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Professor R. A. Sampson, M.A., D.Sc., F.R.S., PRESIDENT, in the Chair.

Lieut.-Commander Harold Dreyer Warburg, R.N., 34 Woodstock Avenue, Golder's Green, N.W.,
was balloted for and duly elected a Fellow of the Society.

The following candidate was proposed for election as a Fellow of the Society, the name of the proposer from personal knowledge being appended:—

Lieut.-Colonel W. A. Tilney, 17th Lancers, Sutton Bonington, Loughborough, Leicestershire (proposed by E. A. Reeves).

Fifty-eight presents were announced as having been received since the last Meeting, including amongst others:—

Portrait of the late Gen. J. F. Tennant (framed drawing), presented by the Executors; L. Silberstein, Vectorial Mechanics; and The Theory of Relativity, presented by the Author; Royal Observatory, Greenwich, Photographic Magnitudes of Stars brighter than 9^m.0, between declination +65° and +75°, presented by the Astronomer Royal.

A Correction to the Determination of the Constants of the Node, the Inclination, the Earth's Ellipticity, and the Obliquity of the Ecliptic. By Ernest W. Brown, M.A., Sc.D., F.R.S.

In my paper with a similar title,* I stated in a footnote that I had not been able to verify Newcomb's statement as to the values for the obliquity which had been actually used in the *Nautical Almanac* for the places of the Moon. Dr. A. W. M. Downing has kindly pointed out to me in a letter that I had overlooked a note † of his in which he had given the required information. Newcomb stated ‡ that the value of Bessel was used from 1847 to 1862, that of Leverrier from 1863 to 1900, and that of Newcomb after 1900; the fact is that the value of Hansen was used from 1862 to 1874, then that of Leverrier to 1900. The change in my work is therefore in the period 1862 to 1874.

The values in question are

$$\text{Hansen, } 23^{\circ}27'31''.42 - 46''.7839(t_c - 0.5), §$$

$$\text{Newcomb, } 23^{\circ}27'31''.68 - 46''.84(t_c - 0.5),$$

where t_c is the number of centuries reckoned from 1800. Hence the correction to reduce Hansen's value to Newcomb's, my adopted standard, is

$$+ 0''.26 - 0''.06(t_c - 0.5)$$

for the years 1862 to 1874, instead of

$$- 0''.27 - 1''.14(t_c - 0.5)$$

in 1862, and

$$- 0''.15 + 0''.75(t_c - 0.5)$$

from 1863 to 1874, applied in § 11 of my paper.

In consequence, the following *additions* must be made to the columns headed Obl. and Br. in Tables II., III. :—

Additions to

TABLE II.

Per.	Obl.	Br.
99	-55	-5
100	-28	-3
101	-20	-2
102	-10	-1
103	+ 1	0
104	+10	+1
105	+19	+2
106	+23	+2
107	+24	+3
108	+21	+2
109	+16	+2
110	+ 6	+1

TABLE III.

Per.	Obl.	Br.
99	- 3	0
100	-13	-1
101	-21	-2
102	-26	-2
103	-27	-2
104	-24	-2
105	-19	-2
106	-10	-1
107	- 1	0
108	+ 8	+1
109	+15	+2
110	+14	+2

* "The Determination of the Constants of the Node, the Inclination, the Earth's Ellipticity, and the Obliquity of the Ecliptic from the Greenwich Meridian Observations of the Moon, 1847-1901," *M.N.*, vol. lxxiv. pp. 552-568.

† *Amer. Eph. Papers*, vol. ix. p. 47.

‡ *M.N.*, vol. lxix. p. 618.
§ *Tables de la Lune*, p. 45.

In § 15, the new addition to the value of F is

$$+ 0''.4 - 0''.8t_c,$$

and in § 16, the correction to the coefficient of $\sin F$ becomes $+ 0''.08$ instead of $+ 0''.07$.

The term of period $28P$ becomes even more doubtful than before, and I shall omit the results based on its possible existence.

The terms with argument Ω which were obtained as the coefficients of products of $\cos F$, $\sin F$ with $\cos \Omega$, $\sin \Omega$ in three cycles, are changed to the following, after the transformation to four periodic terms with arguments $F \pm \Omega$:—

	I.	II.	III.	Mean.
$\cos (F + \Omega)$	$+ 0''.38$	$+ 0''.40$	$+ 0''.16$	$+ 0''.31$
$\sin (F + \Omega)$	$+ 0''.07$	$- 0''.04$	$+ 0''.16$	$+ 0''.06$
$\cos (F - \Omega)$	$- 0''.04$	$+ 0''.04$	$- 0''.04$	$- 0''.01$
$\sin (F - \Omega)$	$- 0''.01$	$+ 0''.07$	$- 0''.24$	$- 0''.06$

The coefficient of $\sin (F + \Omega)$ is now analysed into a term

$$0''.00 + 0''.25(t_c - 0.5),$$

instead of $- 0''.12 + 0''.44(t_c - 0.5)$, while the coefficient of $\cos (F + \Omega)$ becomes $0''.46 - 0''.60(t_c - 0.5)$.

In the discussion and final results, some slight changes must be made. The observed coefficient of the principal term in latitude (§ 21) requires an addition of $0''.01$, which makes it

$$18461''.40.$$

To the observed longitude of the node must be added $- 0''.4 + 0''.8t_c$. The observed value is then

$$33^\circ 16' 27''.86 - (5'' + 134^\circ 08' 46''.18)t_c + 7''.47t_c^2.$$

Newcomb has made the same error in his paper, but the changes in his results will be somewhat smaller because the observations he uses are spread over a longer interval of time. But the difference is not large enough to impair the general agreement of my values with his.

The observed centennial motion of the node, epoch 1850, is now $- 6967943''$, which is $10''$ numerically greater than the theoretical value (§ 25).

The observed coefficient of $\sin a$ (§ 27) becomes

$$- 8''.31 - 0''.25(t_c - 0.5).$$

At the mean epoch, 1875, this is equal to $- 8''.37$.

In § 28, the new centennial motion of the obliquity, as derived from meridian observations of the Moon, is $- 47''.09$, which is very close to that derived by Newcomb from the occultations when his results have been corrected for the same error.

The ratios of the residuals of the mean motions of the perigee and node and the coefficient of $\sin a$ to the portions of them depending on the ellipticity are now (epoch 1850)

$$+ \frac{17''}{638''}, \quad \frac{- 10''}{- 597''}, \quad \frac{0'' \cdot 0}{8'' \cdot 3}$$

or $+0 \cdot 27$, $+0 \cdot 17$, $0 \cdot 00$, respectively (§ 31). The value of the ellipticity deduced from the mean of the first two is now given by

$$\frac{1}{k} = 293 \cdot 7,$$

a little closer than before to the Cape-Greenwich determination, $294 \cdot 4$. If $294 \cdot 0$ be adopted, the correction to the obliquity deduced from the third ratio is $+0'' \cdot 17$.

The total correction to the obliquity (§ 32) is now $+0'' \cdot 17 - 0'' \cdot 25(t_c - 0 \cdot 5)$. This added to the adopted value makes the final value, deduced from the meridian observations of the Moon,

$$23^\circ 27' 31'' \cdot 85 - 47'' \cdot 09(t_c - 0 \cdot 5).$$

The remaining discussion in §§ 33–35 is not affected, since the only change is the reduction of the term in latitude, ascribed to the irregularities of the limb, from $-0'' \cdot 37 \cos a$ to $-0'' \cdot 31 \cos a$.

Yale University:
1915 March 8.

The Elements of the Moon's Orbit. By Ernest W. Brown,
M.A., Sc.D., F.R.S.

1. In a series of papers published in the *Monthly Notices* * during the past two years, I have, building largely on the work of my predecessors in this line, G. B. Airy and P. H. Cowell, compared the Greenwich meridian observations with my theory of the Moon's motion, mainly for the purpose of finding values for the elements to be used in the new lunar tables. The investigations showed that these observations gave the required constants with an accuracy surpassing that furnished by the occultations even when the latter had been treated by so great a master as

* "The Longitude of the Moon from 1750 to 1910," vol. lxxiii. pp. 692–714. Referred to below as the "Longitude" paper. "The Mean Latitudes of the Moon and Sun," vol. lxxiv. pp. 156–167. "The Perigee and Eccentricity of the Moon, 1750 to 1901," vol. lxxiv. pp. 396–424. Referred to below as the "Perigee" paper. "The Determination of the Constants of the Node, the Inclination, the Earth's Ellipticity, and the Obliquity of the Ecliptic from the Greenwich Observations of the Moon, 1847–1901," vol. lxxiv. pp. 552–568. Referred to as the "Node" paper. "A Correction to the Determination of the Constants of the Node, the Inclination, the Earth's Ellipticity, and the Obliquity of the Ecliptic," in this number. Referred to as the "Corrected Node" paper.

Simon Newcomb.* There is an exception to this statement in the mean motion, and this exception arises solely from the fact that the available occultations extend a hundred years further back than the meridian observations.

2. In this final paper on the subject I gather together the results and set forth the values of the elements which seem to be best suited for adoption in the tables, giving briefly the reasons for each choice. The point of view is mainly that of satisfying the modern observations (that is, those for the past two or three centuries) with a consistent set of elements. Empirical terms are avoided as far as possible, and theoretical rather than observational values are chosen for those constants which can be found from theory when the elements are known.

The principal examples of the latter are the mean motions of the perigee and node; their theoretical and observed values have been reconciled by the adoption of the value $1/294$ for the Earth's ellipticity, a value which also reconciles the best observational determination of the lunar parallax with its gravitational value. Since the only part of metrical astronomy in which an exact value of this constant is important is that pertaining to the place of the Moon, no difficulties will be caused by the change.

The most difficult choice is that to be made for the secular acceleration of the mean longitude; the reasons for the adoption of the theoretical value are discussed below. A single empirical term, the large one with a period of some two and a half centuries, is inserted.

3. In the "Node" paper it was shown that the new value for the motion of the obliquity of the ecliptic determined from the Greenwich meridian observations of the Moon agreed very closely with that found by Newcomb from the occultations, and it seems probable that the complete determination of the obliquity from the Moon is rather more accurate than from the Sun, as Newcomb believed. But the longitude, latitude, and parallax, as found by theory, are not affected by any small change in this angle. It is only in the transformation to right ascension and declination that the exact obliquity is required, and here it seems advisable that the adopted value should be the same as that used for the Sun and planets. In any case, the change from Newcomb's solar value at the present time is nearly zero, the addition to it found in my "Corrected Node" paper being $+0''\cdot17 - 0''\cdot25T_1$, where T_1 is the number of centuries from 1850.0.

4. I have followed Newcomb's plan and notation in giving the three principal angles at the epochs 1800, 1850, 1900.† The values of all constants which have been used are given below either numerically or by reference.

* In "Researches on the Motion of the Moon, part ii," *Amer. Eph. Papers*, vol. ix. part i. Referred to below as "Researches II."

† A small error occurs in Newcomb's value ("Researches II," p. 224) for the mean longitude at 1900.0. The seconds of the constant term should read $13''\cdot48$ instead of $14''\cdot72$.

5. *Elements of the Moon's Orbit.*— T_0, T_1, T_2 represent the number of centuries, each 36,525 days, from 1800.0, 1850.0, 1900.0 respectively. There are 18,262 days in each of the two successive intervals between the three epochs. The origins of reckoning are the mean equinoxes of the respective dates. The letter r represents a complete revolution of 360° .

Mean Longitude.

$$\begin{aligned} 1800 & 335^\circ 43' 27'' \cdot 81 + 1336^r 307' 53'' \cdot 11 \cdot 80 T_0 + 7'' \cdot 12 T_0^2 + 0'' \cdot 0068 T_0^3 \\ 1850 & 123^\circ 04' 47'' \cdot 98 + 1336^r 307' 53'' \cdot 18 \cdot 92 T_1 + 7'' \cdot 13 T_1^2 + 0'' \cdot 0068 T_1^3 \\ 1900 & 270^\circ 26' 11'' \cdot 71 + 1336^r 307' 53'' \cdot 26'' \cdot 06 T_2 + 7'' \cdot 14 T_2^2 + 0'' \cdot 0068 T_2^3 \\ & \text{Empirical term, } + 10'' \cdot 71 \sin (240^\circ \cdot 7 + 140^\circ \cdot 0 T_2). \end{aligned}$$

Longitude of Perigee.

$$\begin{aligned} 1800 & 225^\circ 23' 47'' \cdot 81 + 11^r 109^\circ 03' 16'' \cdot 72 T_0 - 37'' \cdot 03 T_0^2 - 0'' \cdot 045 T_0^3 \\ 1850 & 99^\circ 51' 56'' \cdot 38 + 11^r 109^\circ 02' 39'' \cdot 66 T_1 - 37'' \cdot 10 T_1^2 - 0'' \cdot 045 T_1^3 \\ 1900 & 334^\circ 19' 46'' \cdot 40 + 11^r 109^\circ 02' 02'' \cdot 52 T_2 - 37'' \cdot 17 T_2^2 - 0'' \cdot 045 T_2^3 \end{aligned}$$

Longitude of Node.

$$\begin{aligned} 1800 & 33^\circ 16' 27'' \cdot 86 - 5^r 134^\circ 08' 46'' \cdot 17 T_0 + 7'' \cdot 46 T_0^2 + 0'' \cdot 008 T_0^3 \\ 1850 & 146^\circ 13' 41'' \cdot 95 - 5^r 134^\circ 08' 38'' \cdot 70 T_1 + 7'' \cdot 47 T_1^2 + 0'' \cdot 008 T_1^3 \\ 1900 & 259^\circ 10' 59'' \cdot 79 - 5^r 134^\circ 08' 31'' \cdot 23 T_2 + 7'' \cdot 48 T_2^2 + 0'' \cdot 008 T_2^3 \end{aligned}$$

Eccentricity, $\cdot 054900489$,

corresponding to the coefficient $22639'' \cdot 550$ of the principal elliptic term.

Sine half inclination, $\cdot 044886967$,

corresponding to the coefficient $18461'' \cdot 400$ of the principal term in latitude.

Ratio of Earth's to Moon's mass, $\frac{E}{M} = 81 \cdot 53$.

*Ratio of the equatorial radius of the Earth to } $3419'' \cdot 437$,
mean distance of Moon.*

corresponding to the coefficient $3422'' \cdot 540$ of the constant term in the sine of the Moon's equatorial horizontal parallax.

Characteristic of parallactic terms, $\frac{a}{a'} \cdot \frac{E-M}{E+M} = \cdot 00251287$.

The constants of the Earth's orbit are those adopted in Newcomb's Tables of the Sun, except the solar parallax, which is taken to be $8'' \cdot 806$.

The precession is Newcomb's value.

The adopted ellipticity of the Earth's figure is $1/294 \cdot 0$.

The constants of the planetary masses and orbits are those given on p. 31 of part v. of my "Theory of the Moon's Motion."*

* *Mem. R.A.S.*, vol. lix.

Discussion on the Adopted Elements.

6. *The Mean Longitude.*—It is a difficult matter to decide on the form to be given to this function. There are several questions which must be considered, all of them more or less related. In the first place, the observational value of the secular acceleration, depending mainly on ancient eclipses, is by no means a settled quantity, different investigators deriving different amounts, none of which agree with the theoretical value sufficiently closely for adoption. Second, the observations of the past two and a half centuries cannot be represented without the introduction of a large empirical term in the longitude for which no explanation has yet been accepted, and the constants of this term, as deduced from observation, partly depend on the value adopted for the secular acceleration. Finally, the observations of at least the last century are not represented without the introduction of some additional empirical corrections which do not seem to be capable of being represented permanently by harmonic terms, and whose sources are still only in the speculative stage.

The relation of the secular acceleration to the great empirical term is such that either may be changed within limits without affecting the representation of the modern observations (*i.e.* those since 1600) outside the probable errors. We can, in fact, settle the amount of the secular acceleration independently, and then so alter the longitude at epoch, the mean motion, and the great empirical term that the observations are still well represented. There is little question that the minor fluctuations must be left aside and excluded from the tables, since no prediction of their future values can be made with any confidence. The periods are sufficiently short in comparison with that of the great empirical term not to affect the determination of the latter.

7. Before deciding on the value to be adopted for the secular acceleration, it is necessary to consider the various ways in which the tables are used. Their principal scientific purpose is the prediction of the Moon's place for the almanacs, in order that comparison with observation may furnish further data for the investigation of the differences from theory. Another important use is the accurate prediction of solar eclipses needed by astronomers in other fields of work. For the former, it is advisable that the tables should possess the least possible degree of empiricism; for the latter, predictions of the minor fluctuations should be inserted, even though these can only be extrapolated from their run in the few previous years. There are still the needs of those who investigate observations made a thousand or more years in the past to be considered. These conflicting interests can, I believe, be best served in the following way.

8. The tables should be constructed with the theoretical value of the secular acceleration: the great empirical term, the longitude at epoch, and the mean motion having been so determined as best to satisfy the modern observations. The hourly predictions of

the Moon's place should be computed from the tables thus constructed without alteration.

The eclipse predictions should be computed in the same manner, but an empirical addition, carefully stated, should be made to the mean longitude and to the principal arguments depending on it, this addition being deduced from the observations of the few previous years. As the calculations for the almanacs are made some four years ahead, and as the available data for prediction of the minor fluctuations ends a year or two further back, it will be advisable to append to the eclipse predictions the changes arising from $1''$ of change in the mean longitude. It will then be possible, a few months preceding the date of the eclipse, to use data for prediction of the error of the tables ending at most two years before the occurrence. With meridian observations or occultations only averages over several months will give good results, these on any one night or group of nights being subject to systematic error owing to irregularities of the limbs producing apparent inequalities of short period. The Harvard photographic positions need practically no correction for this defect, because the apparent errors of the Moon's centre due to this cause are nearly constant and are the same as those actually present in the eclipse. Hence the necessary data may be obtained in this way much nearer the time of the eclipse.

For the investigation of ancient eclipses a few short and simple additional tables will suffice. These will give the changes in the principal arguments due to a change of $1''$ in the mean motion in longitude, those due to a change of $1''$ in the secular acceleration in longitude, and those due to a change of $1''$ in the secular acceleration of the node.

9. There is another reason for adopting theoretical values wherever possible. In his investigation of ancient eclipses, Dr. Cowell not only deduces an acceleration of the mean motion, but also one of the angle between the Sun and the Moon's node. He attributed this at first to the node, but on Newcomb objecting that the latter has its acceleration well determined, he suggested that it could equally well be attributed to the Sun. As a matter of fact, so long as the cause of the additional acceleration of the Moon's longitude is in doubt, it is equally possible that there may be also an additional acceleration of the node. With the two known causes which do produce such changes, the accelerations of the node are of the same order of magnitude as those of the mean longitude, and, in the absence of an accepted cause, there is no more reason for excluding the node from such an addition than the mean longitude. The simplest way out of the difficulty is to exclude both and leave the whole question open.

10. It remains, then, to give the values of the mean longitude and great empirical term which best satisfy the modern observations when the theoretical value of the secular acceleration is adopted.

In my "Longitude" paper it was shown that the value which

Newcomb obtained for the mean longitude in his last researches substantially satisfies the Greenwich meridian observations since 1750.

Hansen's mean longitude is

$$335^{\circ}43'26''.70 + 1336^{\circ}307'53'39''.61T_0 + 13''.30T_0^2 + 0''.0135T_0^3.$$

The correction to this which has been used in the *Nautical Almanac* since 1883, and with which the earlier observations were reduced, is

$$-1''.14 - 29''.17T_0 - 3''.76T_0^2.$$

In his "Researches II.," p. 224, Newcomb obtained a further correction,

$$+0''.83 + 2''.60T_0 - 0''.47T_0^2 - 0''.0067T_0^3,$$

where the last term is alone due to a correction of the theory.

His resulting mean longitude is

$$335^{\circ}43'26''.39 + 1336^{\circ}307'53'13''.04T_0 + 9''.07T_0^2 + 0''.0068T_0^3.$$

But Newcomb omitted one or two sensible planetary terms of very long period which, if included, would have impaired slightly the agreement with modern observations: the earlier observations are not sufficiently accurate to be affected. The neglected planetary terms may be included by adding to the mean longitude

$$-0''.83 - 0''.67T_0 + 0''.44T_0^2,$$

so that the expression for it becomes

$$335^{\circ}43'25''.56 + 1336^{\circ}307'53'12''.37T_0 + 9''.51T_0^2 + 0''.0068T_0^3.$$

With this value must be taken Newcomb's last expression for the great empirical term:

$$+12''.95 \sin (131^{\circ}.0T_0 + 100^{\circ}.6) \quad . \quad . \quad (A)$$

11. It is desired now to change the acceleration to its theoretical value. My computation gave, at 1800, $6''.01 + 1''.11$, the latter being the acceleration of the equinox. Hence the observed value is $2''.39$ greater than that given by theory.

It is shown in § 12 below that we can put

$$(A) + 2''.39T_0^2 = (B) + 2''.25 - 0''.57T_0,$$

where the new empirical term is

$$+10''.71 \sin (140^{\circ}.0T_0 + 100^{\circ}.7) \quad . \quad . \quad (B)$$

and the final mean longitude

$$335^{\circ}43'27''.81 + 1336^{\circ}307'53'11''.80T_0 + 7''.12T_0^2 + 0''.0068T_0^3.$$

We can always alter the longitude at epoch and the mean motion, since they are arbitrary constants of the theory. These,

with the three constants of the empirical term, give five constants which are available for the satisfaction of the observations. The five constants are used to make the two sets of expressions for the longitude agree at five equidistant dates, fifty-one years apart, the central date being 1826.1. The differences at other dates are all less than $0''.1$ between 1720 and 1930, and less than $0''.6$ back to 1625.

12. The transformation is more conveniently made with the following formulæ than with those which I gave in § 17 of my "Longitude" paper. If

$$ct^2 + a \sin mt + b \cos mt = u + vt + x \sin zt + y \cos zt$$

at the five equidistant dates $t = 0, \pm 1, \pm 2$, then, putting $n = \cos m - 1$, we obtain

$$\begin{aligned} \cos z - 1 &= \frac{bn^2}{c + bn}, & x \sin z &= \frac{c + bn}{bn} a \sin m, & y &= \frac{(c + bn)^2}{bn^2}, \\ u &= -c \frac{c + 2bn}{bn^2}, & v &= -\frac{ac}{bn} \sin m. \end{aligned}$$

With $t = 0$ at 1826.1, and fifty-one years as the unit of t , I find

$$\begin{aligned} (A) + 2''.39(T_0 - .261)^2 &= 0''.62t^2 + 12''.95 \sin(66^\circ.8t + 134^\circ.7) \\ &= 1''.94 - 0''.93t + 10''.71 \sin(71^\circ.5t + 137^\circ.2) \\ &= 2''.41 - 1''.82T_0 + 10''.71 \sin(140^\circ T_0 + 100^\circ.7). \end{aligned}$$

Also

$$2''.39(T_0 - .261)^2 = +0''.16 - 1''.25T_0 + 2''.39T_0^2.$$

In this transformation the change is made at the central date. This fact and the altered value of c account for the differences between these results and those obtained in my "Longitude" paper.

13. *The Longitude of Perigee.*—In § 25 of my "Perigee" paper I obtained an expression for the longitude of the Moon's perigee which represented the Greenwich meridian observations in a very satisfactory manner. In this the theoretical secular acceleration of the perigee was adopted. Its centennial mean motion is,* however, found to be $16''$ greater than its theoretical value. The value of the Earth's ellipticity adopted in the calculations was $1/297$. If this be changed to $1/294$ (see § 17 below) the remaining difference is $3''$. The theoretical value is uncertain by at least this amount, because the constants of the Moon's figure are not known with sufficient accuracy, as I have mentioned in computing the effect.† Also the calculations of some portions were only taken to the nearest $1''$, and there may be an accumulation on this account either way of $1''$ or $2''$. Hence the observed value represents theory within the limits of error of the latter.

* After some slight changes due to the change of e , γ , this motion is now found to be $14643520''$.

† *Mem. R.A.S.*, vol. lix. p. 81.

To the value given in my "Perigee" paper I add

$$-0''\cdot045(T_0 - 0\cdot5)^3,$$

the theoretical portion of the secular acceleration depending on t^3 and computed for 1850.0. With this addition, we obtain the value for 1800.0, and thence those for 1850.0, 1900.0, given in § 5 above.

14. *The Longitude of the Node.*—This quantity was determined from the Greenwich meridian observations in the interval 1847 to 1901, the theoretical secular acceleration being used. The agreement with Newcomb's determination from a period four times as long but with a lower degree of accuracy was very close. To the value in § 22 of my "Node" paper is to be added the correction $-0''\cdot4 + 0''\cdot8T_0$ found in my "Corrected Node" paper, and the term $0''\cdot008(T_0 - 0\cdot5)^3$, the additional theoretical portion of the secular acceleration. This gives what may be called the observed value. When the ellipticity is corrected to the value $1/294$, we must add $-12''$ T to the theoretical value* of the motion of the node. This brings the latter within $3''$ T of the observed value, which can, as remarked in the case of the perigee, be used as representing either, since the difference is less than the possible error of the theoretical determination for the same reasons. The value in § 5 above is therefore the observed value in my "Node" paper, with the correction $-0''\cdot4 + 0''\cdot8T_0 + 0''\cdot008(T_0 - 0\cdot5)^3$.

15. *The Eccentricity.*—The coefficient of the principal elliptic term was found from the Greenwich meridian observations to be $22639''\cdot542$ from 1750 to 1850, and $22639''\cdot549$ from 1847 to 1901. It is convenient to have the third decimal place a zero, it having little significance with a probable error of $0''\cdot02$. This coefficient is therefore taken to be $22639''\cdot550$, and is equated to

$$2e - \frac{1}{4}e^3 + \frac{5}{96}e^5 + \frac{107}{4608}e^7,$$

from which the given value of e is obtained.

16. *The Inclination.*—The value determined in § 21 of my "Node" paper for the coefficient of the principal term in latitude was found from the Greenwich observations 1847–1901. To this must be added $0''\cdot01$ obtained in my "Corrected Node" paper. The resulting coefficient is $18461''\cdot40$, with a probable error $\pm 0''\cdot03$. Newcomb's value is $0''\cdot04$ greater, with a probable error $\pm 0''\cdot19$. I adopt the former, and equate it to

$$2\gamma\left(1 - e^2 + \frac{7}{64}e^4 - \frac{5}{288}e^6\right) - \frac{1}{4}\gamma^5(1 - e^2) - \frac{3}{8}\gamma^7,$$

from which, e being known, γ has been obtained.

17. *The Ellipticity of the Earth.*—In the "Node" paper it was shown that the value $1/294$ reconciled the observed and theoretical

* Before this correction is applied, the theoretical value is $-6967934''$ per century at 1850.0, after the change to the finally adopted values of e , γ .

motions of the perigee and node, as also the last Cape-Greenwich determination of the lunar parallax with its gravitational value (see § 19 below).

18. *The Mass of the Moon*.—The best determination of this constant is probably that of Hinks, who finds for the ratio E/M the value $81.53 \pm .05$.* It may be mentioned that in giving this he also summarises and brings up to date the various determinations discussed by Newcomb in his *Astronomical Constants*.

19. *The Parallax of the Moon*.—In "Researches II.," p. 42, Newcomb finds, with the latest values of the Earth's mean radius and of g , that the sine of the equatorial horizontal parallax of the Moon is equal to

$$3418''.65(1 + \frac{1}{3}k),$$

where k is the Earth's ellipticity. This is known as the gravitational value.

In finding it, he has taken 1.0009070 as the ratio of the constant term in the final expression for the inverse of the Moon's radius vector to $1/a$, where $n^2a^3 = E + M$. For this ratio I find from my theory 1.0009076 . The corresponding correction is $+0''.002$.

He has used $E/M = 81.45$. The value adopted here is 81.53 . The correction for this change is $+0''.014$.

The portion in the expression depending on k , when $1/k = 294.0$, is $+3''.876$.

Adding these, we obtain for the required constant

$$3422''.54,$$

which is the value adopted.

This differs from the gravitational value obtained by Dr. Crommelin† in his paper on the discussion of the last Cape-Greenwich series of observations by $0''.02$ when the same ellipticity is used, but it agrees exactly with the value which he found from observation.

The corresponding value of the Earth's equatorial radius divided by a is $3419''.437$.

20. *The Remaining Constants*.—No special mention of these is necessary. The solar parallax, as is well known, is that of Hinks. It is only of importance in the parallactic terms which have powers of a_1 in their characteristics.

* *M.N.*, vol. lxx. p. 74.

† *M.N.*, vol. lxxi. p. 540.

Note on the Solution of Hill's Equation. By Sir J. Larmor, F.R.S.

The differential equation treated by G. W. Hill for the purposes of the Lunar Theory is so important also for other branches of mathematical physics* that it is desirable to bring into view in what respects, if any, the modern abstract theory of linear equations with periodic coefficients can add to or modify Hill's original analysis. The exposition by Mr. Lindsay Ince in the *Monthly Notices* for March (p. 436) allows perhaps, owing to its succinctness, an immediate answer to be given to this question.

The type of solution worked out by Mr. Ince is

$$y = e^{\mu z} u$$

where

$$u = \sin(z - \sigma) + a_3 \cos(3z - \sigma) + a_5 \cos(5z - \sigma) + \dots \\ + b_3 \sin(3z - \sigma) + b_5 \sin(5z - \sigma) + \dots$$

wherein μ and σ are constants.

This is the real part of another solution, the coefficients in the differential equation being real,

$$y = e^{\mu z} \{ -\iota e^{\iota(z - \sigma)} + a_3 e^{\iota(3z - \sigma)} + a_5 e^{\iota(5z - \sigma)} + \dots \}$$

or, employing different complex constants and writing ν for $\mu + \iota$, in which, however, there seems to be no reason why μ should be always real,

$$y = A_0 e^{\nu z} + A_1 e^{\iota(\nu+2)z} + A_2 e^{\iota(\nu+4)z} + \dots$$

The solution developed by Hill, as summarised in view of physical applications by Rayleigh (*loc. cit.*), is of type

$$y = b_0 e^{\nu z} + b_1 e^{\iota(\nu+2)z} + b_2 e^{\iota(\nu+4)z} + \dots \\ + b_{-1} e^{\iota(\nu-2)z} + b_{-2} e^{\iota(\nu-4)z} + \dots$$

in which, so long as ν comes out real, the coefficients are all real quantities.

This latter form leads to a system of linear equations determining the coefficients, and a corresponding determinant determining ν , which are infinite both ways. The previous form, employed by Mr. Ince, with the value $-\iota e^{-\iota\sigma}$ for A_0 , leads to a system infinite only towards the positive side, in which, however, the coefficients are always complex. This scheme, which Mr. Ince works out in trigonometrical form, may be advantageous for the determination of the coefficients in non-astronomical applications for which ν is usually complex. The expansion set out by Mr. Ince for real values of μ and σ belongs to the special case when the root ν of the determinantal equation comes out of the form $1 - \iota\mu$.

* Cf. Rayleigh, *Phil. Mag.*, xxiv., 1887; *Collected Papers*, vol. iii. p. 1.

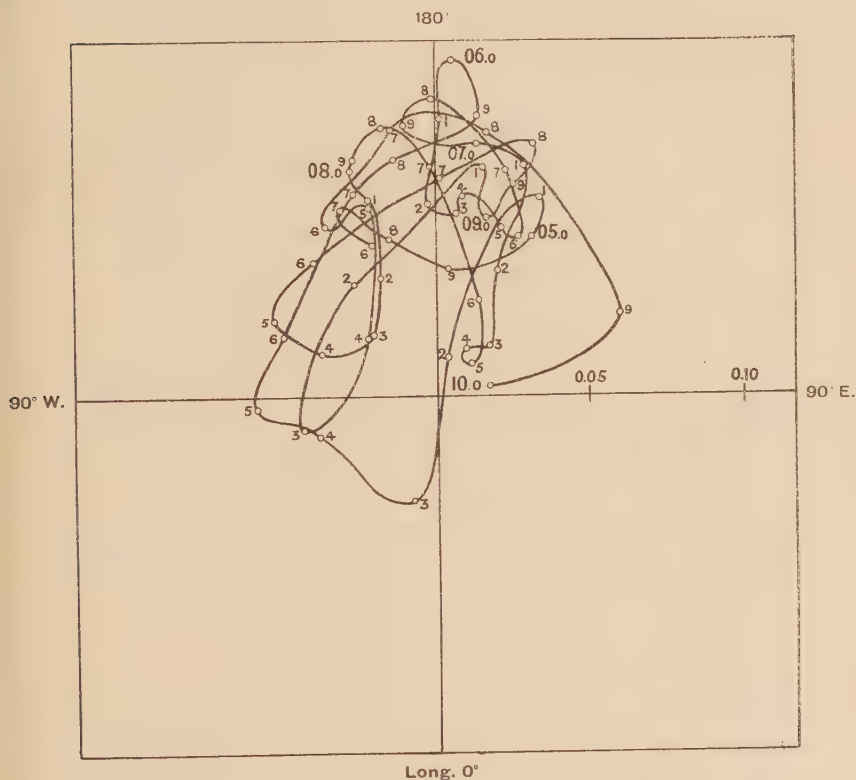
The Irregular Movement of the Earth's Axis of Rotation: a Contribution towards the Analysis of its Causes. Part II.
By Sir Joseph Larmor, F.R.S., and Col. E. H. Hills, C.M.G.,
R.E., F.R.S.

In *Monthly Notices, R.A.S.*, Nov. 1906 (lxvi. p. 22), we considered the graph of the minute wanderings of the Earth's axis of rotation over its surface, as published from year to year under the auspices of the International Polar Commission, with a view to ascertaining something of the terrestrial origins of the irregularities.* An undisturbed free precession would make the pole trace on the Earth a small circle (actually a few yards in radius) uniformly in about 428 days. The principle of our procedure was that, if the actual graph of the polar motion with respect to the Earth's surface is changed to a graph with respect to axes of co-ordinates travelling round, over the surface of the Earth, uniformly with the velocity of this free precession, then the velocity of description, by the pole, of the graph so obtained, multiplied by the Earth's moment of inertia, represents in direction and magnitude the disturbing torque acting on the solid Earth at that instant. Such a torque could arise from change of position and from rate of motion of loose material, supported by the solid Earth, but for the purpose of formation of dynamical equations not counted in its inertia. The succession of torques exerted by the loose material is thus represented by the hodograph of the path of the pole relative to the new axes moving round with the precession: and the form of that curve was accordingly traced.

At each observing station it is the course of *change* in latitude that is determined; and that information, by comparison for the various stations, determines the course of *movement* of the pole of rotation. But the position of the Earth's pole of inertia is not determined on the diagram. It is presumably a nearly central point, chosen to make the precession as nearly circular as possible. Changing it to another point is equivalent to merging a certain shell of the solid Earth with the mobile material, that shell, namely, which must be shaved off to make the remaining Earth dynamically symmetrical around the new axis thus postulated. The difference produced in the torque diagram will be that the torque arising from this shell will now be included in it; but this torque is invariable with respect to the Earth, so that its inclusion amounts merely to changing the origin in the vector diagram of torque. Thus the origin in that diagram also is indeterminate, depending on what surface deposits we choose to include with the solid Earth, as we had previously seen (*loc. cit.*, p. 31); and the natural course is to place it as centrally as possible.

* Cf. also *Proc. Roy. Soc.*, lxxxii. (1909), p. 89; *Monthly Notices, R.A.S.*, Jan. 1915; *Proc. Lond. Math. Soc.*, April 1915.

The diagram of torque for the period 1905-10, here annexed, is constructed on the same principles as the former ones (*Monthly Notices*, lxvii. p. 29). Owing to the revision of the deduced positions of the pole since the diagram for the period 1900-05 was constructed, the last two points on the earlier diagram, *i.e.* those for 1904·8 and 1904·9, must now be erased, and the new positions as given on the present diagram substituted.



Torque Diagram of the Earth's Axis, Years 1905-1910.

It will be seen that an inspection of the directions and magnitudes of the torques during 1907 does not lend support to the view advanced by H. J. Swiers (*Proc. R. Acad. Amsterdam*, xiv. p. 211) that there was then some special cause of perturbation of the regular motion of the pole. The disturbing torques during that year were not of any exceptional magnitude, nor were they changing rapidly. To judge by the diagram, 1907 was a much quieter year than 1905 or 1909. But if recent abnormal discrepancies in the Moon's mean motion really mean a change in the length of the sidereal day, the terrestrial displacements to

which that must be due, by altering the moments of inertia, presumably in a manner not symmetrical, may be expected to show to some extent in the polar movement also.

Would the regular free precession by itself, irrespective of the changes of moments of inertia by which it is excited, have sensible influence on the mean length of the sidereal day? In the most general Poincot case the axis of rotation describes a cone round the fixed axis of angular momentum. The effective mean rotation of the Earth is represented closely by the component angular velocity around this mean axis, which is easily seen to be $2T/G$, where T is the kinetic energy and G the constant angular momentum.* A disturbance, when the axis of rotation is that of maximum moment of inertia, requires an increase of energy, while frictional agencies steadily diminish it: the effect of each is included in the formula. Let us follow out the case of symmetry, in which A and B are equal; and let $G = C\Omega$. The effective average rotation of the Earth, that about the axis of G , is $\omega_3 + \omega_1 \frac{A\omega_1}{C\omega_3}$, approximately, where $C^2\omega_3^2 + A^2\omega_1^2 = C^2\Omega^2$; this agrees with the exact general expression $2T/G$ above, and is approximately $\left(1 + \frac{1}{2}\frac{\omega_1^2}{\Omega^2}\right)\Omega$. The establishment of a regular free precession would thus *increase* the effective angular velocity of the Earth; but if it is of amplitude n seconds of arc the increase would integrate to only $n^2/30$ seconds of time in a century.

We may remark in passing that the energy of the Earth's magnetic distribution is usually supposed to have come somehow out of the Earth's rotation. If it were wholly accounted for by work done by the rotating Earth against some extraneous cosmical resistance, the consequent change in the diurnal angular velocity ω would be given very roughly by an equation

$$\frac{1}{8\pi} H^2 \text{ vol.} = -\delta\left(\frac{1}{2}Ek^2\omega^2\right),$$

in which the Earth is supposed uniformly magnetised so that H is uniform throughout. Taking $H = .5$ c.g.s. and $k^2 = \frac{1}{3}a^2$, this gives

$$\frac{\delta\omega}{\omega} = -25 \cdot 10^{-10},$$

a change which would make the Earth lag by 8 seconds of time in a century. If the fraction $1/n$ of the terrestrial magnetic energy were thus renewed each century, there would be a secular retardation of the Earth's rotation such as would produce a lag

* As follows:—The motion is the Poincot rolling of the momental ellipsoid $Ax^2 + By^2 + Cz^2 = 1$ on a fixed plane; if so, $\Omega = (\omega_1, \omega_2, \omega_3) = \theta(x, y, z)$ when (x, y, z) is its point of contact with the plane; hence $T = \frac{1}{2}\theta^2$ and $G = \theta/p$, where p is the distance of the centre from this plane, which is thus constant while its direction is that of G , the fixity of the plane of rolling being thereby in fact verified. The component angular velocity around p is $\Omega p/r$, which is $2T/G$ as in the text.

of $4/n$ seconds in one century, an amount which, when the rapid motion of the magnetic poles is considered, is perhaps not an absolutely negligible contribution to the lag arising from other causes, especially as the loss of purely magnetic energy would not be the whole loss concerned.

We have recently discovered that the diagram of observed polar displacement has been analysed in a different way in the treatise by F. Klein and A. Sommerfeld, *Theorie des Kreisels*, Heft iii. (1903), p. 676. Their procedure is to eliminate the 428-day periodic part by graphing the difference between the position of the pole at each instant and the position 428 days earlier. This is, of course, done immediately by plotting the line joining the two positions, with regard to a new origin. The result which they thus obtained showed traces of an annual period; and when that also was removed by similar treatment, the residue appeared to be quite irregular. The question suggests itself—can any physical interpretation be attached to this process and to the derived type of graph? It would appear from our previous discussion that in it each radius represents the vectorial time-aggregate of the torques that have been acting during the preceding 428 days, and so hardly admits of comparison with the course of disturbances; indeed that interpretation is obvious, for it represents the total change of angular momentum during that time, as the free precession has completed its period. The second derived graph does not appear to admit of any direct dynamical meaning.

A Determination of the Systematic Motions of the Stars from the Radial Velocities. By A. S. Eddington, M.A., M.Sc., F.R.S., and W. E. Hartley, M.A.

1. The following investigation of the phenomenon of star-streaming is based on the radial velocities published by W. W. Campbell in *Lick Observatory Bulletins*, Nos. 211 and 229. Its object was the determination of the position of the vertex, and the average motion along the three axes of the velocity-ellipsoid, for the different spectral types separately. It was desired to test the accordance with the results derived from proper motions, and to obtain some information as to the comparative behaviour of the different types. It was also of interest to determine whether the motions at right angles to the direction of star-streaming are symmetrical, and in particular whether there is any relation to the plane of the galaxy apart from the main phenomenon of the star-streams.

Before the discussion had progressed beyond a preliminary stage, a paper was published by K. W. Gyllenberg* dealing with the same problem, and it became a question whether we should

* *Lund Meddelande*, No. 59.

proceed with our plan. His work is based on correlation methods, which at first sight appear to differ widely from ours; but actually the differences are mainly in the nomenclature, and the real divergences are confined to one or two details. Our reasons for making a second investigation are twofold: (1) we consider that the weighting in the correlation method is exceedingly faulty, and there is great loss of accuracy from this cause; our own weighting of the observations is by no means ideal, but it seems to represent a substantial gain; (2) we are able to discuss a smaller number of stars and are thus able to exhibit the transition from type to type by smaller steps. Perhaps too the extremely interesting results obtained by Gyllenberg are a sufficient justification for a further examination of the same material.

2. *Theory.*—According to Schwarzschild's ellipsoidal law, the stellar velocities are distributed in such a way that the number of velocities between (u, v, w) and $(u+du, v+dv, w+dw)$ is proportional to

$$e^{-s} du dv dw,$$

where $s=1$ is the equation of an ellipsoid, called the velocity-ellipsoid.

If the ellipsoid referred to its principal axes be

$$s \equiv \frac{u^2}{\lambda_1} + \frac{v^2}{\lambda_2} + \frac{w^2}{\lambda_3} = 1. \quad (1)$$

it can be shown* that V , the mean radial velocity (regardless of sign) in the direction (l, m, n) , is given by

$$V^2 = \lambda_1 l^2 + \lambda_2 m^2 + \lambda_3 n^2.$$

There is, strictly, a numerical factor multiplying V^2 , which, however, we need not consider.

Hence, referred to any axes,

$$V^2 = \alpha l^2 + \beta m^2 + \gamma n^2 + 2\delta ml + 2\epsilon nl + 2\zeta lm. \quad (2)$$

and, when $\alpha, \beta, \gamma, \delta, \epsilon, \zeta$ have been found, the directions and lengths of the axes can be determined by the usual processes of analytical geometry.

The obvious course is to substitute in (2) the observed values of V, l, m, n for each star, and thus obtain a set of equations of condition to be solved by least squares. But there is a serious objection to this course. In any direction the values of V for the different stars are distributed according to the error-law with a modulus proportional to the mean value $\bar{V}$. The probable deviation of any observed quantity V^2 from the mean value $\bar{V}^2$ is thus proportional to $\bar{V}^2$, and the weight of any equation of condition is therefore proportional to $1/\bar{V}^4$. It is found that $\bar{V}$ in the direction of star-streaming is about twice as great as in the directions at right angles, so that certain of the equations ought to have only

* Eddington, *Stellar Movements and the Structure of the Universe*, p. 146.

$\frac{1}{18}$ the weight of others. There must be a great waste of material in weighting all the equations equally, and it is this loss of accuracy (which occurs in the same way in the correlation method of Gyllenberg) that we desire to avoid.

The true weights cannot be determined until the solution has been made, so that we cannot do more than minimise the inequality. Consider the equation

$$V = al^2 + bm^2 + cn^2 + 2fmn + 2gnl + 2hlm, \quad (3)$$

V being taken without regard to sign.

Equations (2) and (3) cannot be made to correspond, but differ by harmonics of the fourth degree in l, m, n . With the prolateness of the velocity-ellipsoid actually found, however, a fairly close agreement can be obtained. Although (2) represents Schwarzschild's law exactly, there is no ground for pressing the accuracy of this law so far as to discriminate between (2) and (3); indeed we know that there are actually terms of the third degree giving a skewness in the distribution. By using (3) we shall determine the ellipsoidal (or æolotropic) character of the distribution of velocities equally as well as if we used (2).*

By using (3) the extreme range of weights is 4:1, instead of 16:1 when (2) is used. It does not seem practicable to equalise the weights further, and we have been content with this improvement. There is the further gain that, since there are always a few excessive velocities, the precise limits of rejection matter less when the velocities are not squared; for when a great velocity is squared, its retention or rejection is likely to have a very great influence on the result.

3. *The Computations.* The values of l, m, n, V have been calculated for each star, and normal equations corresponding to the equations of condition (3) formed for the separate groups discussed. The velocity V is the residual velocity regardless of sign after correcting for the solar motion and for the "K" term (supposed to represent a systematic error in the measured velocities). The values used for the solar motion and K are those determined by Campbell (*L.O.B.*, Nos. 196 and 211) from a somewhat shorter list of radial velocities (except in Type A), the Apex being assumed at R.A. 270° , Dec. $+30^\circ$.

Type.	Solar Motion. Km. per sec.	K. Km. per sec.
A	15.3	-0.6
F	15.8	+0.06
G	13.9	+0.22
K	18.9	+1.91
M	20.2	+4.59

* The presence of small spherical harmonics of the fourth order does not affect the determination of the coefficients of the harmonics of the second order in a least-squares solution if the observed stars are fairly symmetrically distributed; and we could if necessary deduce the values of the coefficients in (2) from our solution of (3). But there appears to be no object in this refinement.

Since the formation of the coefficients of the normal equations is very laborious, a few words may be said as to the most convenient method of computation. The device of making the work of a least-squares solution depend on tables of squares is well known, and in the present problem it can be considerably extended. The coefficients of $a, b, c, 2f, 2g, 2h$ are of the forms $\Sigma l^4, \Sigma l^2 m^2, \Sigma l^3 m, \Sigma l^2 mn$; there are thirty-six altogether, but owing to duplication there are only fifteen different ones. The following scheme was used, each entry being the heading of a column on the computation sheets:—

Sheet 1.	l	m	n	$(m+n)$	$(n+l)$	$(l+m)$	$(l+m+n)$
„ 2.	l^2	m^2	n^2	$(m+n)^2$	$(n+l)^2$	$(l+m)^2$	$(l+m+n)^2$
„ 3.	l^4	m^4	n^4	$(m+n)^4$	$(n+l)^4$	$(l+m)^4$	$(l+m+n)^4$
„ 4.	$(l+m)^2 \sim l^2$	$(l+m)^2 \sim m^2$	$(n+l)^2 \sim n^2$	$(n+l)^2 \sim l^2$	$(m+n)^2 \sim m^2$	$(m+n)^2 \sim n^2$	
„ 5.*	$(m^2+2lm)^2$	$(l^2+2lm)^2$	$(l^2+2ln)^2$	$(n^2+2ln)^2$	$(n^2+2mn)^2$	$(m^2+2mn)^2$	
„ 6.	Vl^2	Vm^2	Vn^2	$V(m+n)^2$	$V(n+l)^2$	$V(l+m)^2$	$V(l+m+n)$

No attention need be paid to sign except in forming $(m+n)$, etc., on the first sheet; multiplication tables are only used for the multiplication by V on sheet 6. The sums of the separate columns on sheets 2, 3, 5, and 6 were taken for all the stars of each type, and the coefficients are then simple linear combinations of the resulting quantities, *e.g.*:—

$$2\Sigma l^2 m^2 = \Sigma(l^2 + 2lm)^2 + \Sigma(m^2 + 2lm)^2 - \Sigma(l+m)^4$$

$$\Sigma l^2 mn = \Sigma mn - \Sigma mn^3 - \Sigma n^3 m.$$

An efficient check is provided by the last column, and especially by

$$\Sigma \Sigma (l^2 + 2lm)^2 + \Sigma \Sigma l^4 = \Sigma \Sigma (l+m)^4 + \text{number of stars,}$$

where the second Σ denotes summation of all the terms of the type indicated.

The normal equations were solved for a, b, c, f, g, h , and the principal axes of the ellipsoid were then found by the use of the discriminating cubic in the usual way.

Rejections.—There is a danger in rejecting stars with large velocities, because these occur more frequently in the direction of the major axis of the ellipsoid, and the effect is to diminish the inequality for which we are looking. Fortunately there are very few velocities in the lists which seem urgently to demand rejection. We have been content with rejecting the following two stars:—

Lal. 5761,	Type F,	Velocity 158 km. per sec.
C.Z. 5 ^h 243,	Type G,	Velocity 229 km. per sec.

Since, however, Type G is exceptional in including several large velocities which might have an undue influence, a second solution for this type was made in which we rejected the star with

* Sheet 5 is formed from sheet 4 by squaring.

the greatest velocity in each of seventeen equal regions into which the sky was divided. This removed all the very large velocities and at the same time assured equal treatment to the different directions. The rejection of the largest 17 errors out of a total of 153 will theoretically diminish the mean error in the ratio $1/1.235$; accordingly the lengths of the resulting axes have been increased in the ratio $1.235 : 1$ to allow for the rejections.

4. *Results.*—The results of the determinations are given in Table I. (page 526). In Table II. are given the results for some alternative divisions of the stars into groups.

Confining attention to Table I., the most interesting result is the determination of the direction of the greatest axis, *i.e.* of the vertex. It will be seen that it agrees closely with the position, R.A. 94° , Dec. $+12^\circ$, derived from the proper motions, in all cases except Type G which deviates strongly. Another result is that the least axis for Type A points to the galactic pole; but in the other types the mean and least axes seem to have no consistent relation to the galaxy, and it is doubtful if the deviations from the spheroid are more than accidental.

Adopting the spheroid as a sufficiently close representation, we have taken the mean of the two smaller axes of the ellipsoid to give the minor axis of the spheroid. The ratio of this to the major axis is given, as the reciprocal of the prolateness, in the last column but one. The increase of the minor axis with progression in type is fairly well marked; this represents the true increase in the *motûs peculiares* free from the effects of star-streaming. The fact that the prolateness is substantially less for Types K and M than for the earlier types is in accordance with what has been found by a number of investigators from the proper motions. Moreover, the actual values found for the prolateness are in general agreement with the results derived from the proper motions.*

Owing to the non-uniform distribution of the different types in the sky and to the progressive change of average speed with type, it is undesirable to make a general solution taking all the types together; such a solution would probably be fallacious. But in order to give a result which may be comparable with determinations made from the proper motions in which the types are not separated, we may take a mean between the different groups in Table I., weighting according to the number of stars in each group. Accordingly the mean result deduced from 1086 stars is:

Vertex,	R.A. $94^\circ.6$, Dec. $+12^\circ.5$
Ratio of axes,	0.59

* Five investigations of different series of proper motions on the ellipsoidal hypothesis have given values of the ratio of the axes ranging from 0.47 to 0.63; these results refer to a mixture of types (*Stellar Movements and the Structure of the Universe*, p. 125). To these may now be added H. Raymond's investigation (*A.J.*, No. 676), with results ranging for different types from 0.49 to 0.61. H. S. Jones (*Monthly Notices*, vol. lxxiv. p. 194) gives the ratio of the axes of the velocity-ellipse for different types near the N. Pole ranging from 0.56 to 0.78; the ratios for the ellipsoid would be slightly less.

TABLE I.

Type.	Greatest Axis.			Mean Axis.			Least Axis.			Spheroid Minor Axis.	Prolate-ness $1 \div$	Number of Stars.
	Km. per sec.	R.A.	Dec.	Km. per sec.	R.A.	Dec.	Km. per sec.	R.A.	Dec.			
A	16.02	98.7	+15.7	9.79	162.8	-56.9	6.88	197.3	+28.3	8.34	.52	212
F	20.84	92.9	+16.8	12.39	245.9	+71.3	9.24	180.4	- 8.0	10.82	.52	199
G	30.03	68.8	-13.5	18.56	181.2	-57.9	11.81	151.3	+28.5	15.18	.51	152
K-K2	20.85	101.3	+19.9	16.93	246.1	+66.1	11.78	186.7	-12.7	14.36	.69	366
K5-M	24.60	100.2	+ 7.7	17.95	191.6	- 9.8	12.72	152.6	-77.4	15.34	.62	157
Vertex	...	94.2	+11.9	...	...	...	...	...	...	...	...	...
Galactic pole	...	...	...	...	...	...	...	190°	+28°	...	...	...

TABLE II.

Type.	Greatest Axis.			Mean Axis.			Least Axis.			Spheroid Minor Axis.	Prolate-ness $1 \div$	Number of Stars.
	Km. per sec.	R.A.	Dec.	Km. per sec.	R.A.	Dec.	Km. per sec.	R.A.	Dec.			
Ao (including Δp)	14.10	51.7	+49.6	11.57	113.9	-22.1	5.67	189.2	+32.0	12.83*	.27†	98
A1-A9	19.93	105.5	+11.9	8.01	+	+	8.01	+	+	8.01	.40	114
G (rejecting 17 stars)	28.46	62.9	- 8.5	16.27	110.7	+77.4	11.37	154.4	- 9.1	13.82	.49	136
K	22.39	106.3	+21.0	17.07	240.0	+60.8	11.60	188.6	-19.1	14.34	.64	443
F and G, giants	16.95	76.7	- 3.7	11.20	166.2	+ 8.8	6.89	189.2	-80.4	9.04	.53	175
F and G, dwarfs	28.44	81.7	+14.9	17.80	117.6	-71.8	8.38	174.4	+10.2	13.09	.45	160
K-K2, distant	18.45	116.1	+43.0	13.14	252.3	+37.7	7.18	182.0	-23.6	10.16	.55	185
K-K2, near	26.60	107.7	- 7.3	20.84	227.7	-75.8	13.52	196.1	+12.1	17.18	.65	181

* Major axis.

† Oblateness.

‡ Indeterminate.

The deviation of the vertex for Type G and the very large corresponding velocity are noticeable features; this is not the only instance of anomalous behaviour on the part of Type G in stellar investigations.* It will be seen that an additional determination given in Table II., in which the very great radial velocities were rejected according to the plan explained above, gives practically the same results. In Raymond's discussion of the Boss proper motions the vertex for Type G is found to be at R.A. $81^{\circ}6$, Dec. $+25^{\circ}5$; the deviation in right ascension is in the same direction, but in declination it is in the opposite direction.

Turning now to Table II., an interesting result is obtained by dividing Type A into earlier and later portions. Type A₀ (including A_p) does not appear to exhibit the phenomenon of star-streaming at all. The two greater axes are nearly equal (14.10 and 11.57 km. per sec.), so that their directions are scarcely determinate; in any case the direction found for the greatest axis is too far from the vertex (as usually adopted) to be any certain indication of star-streaming. But the least axis is well marked and points within 5° of the galactic pole. We may say that the stars of Type A₀ tend to move parallel to the galactic plane, and have no preference for any special direction in that plane. They appear to be associated in character with the previous type, B, (which we have not discussed here, since its behaviour is well known). The other division, A₁–A₉, shows the star-streaming fully developed, and no trace remains of any general tendency to move parallel to the galactic plane. The comparatively small number of stars in these two divisions suggests the need of an independent confirmation of this divergence of behaviour.

5. *Giant and Dwarf Stars.*—Another division has been made of the stars of Types F and G into "giants" and "dwarfs," following Hertzsprung and Russell's classification. No rigorous separation can be made; but the two groups will be fairly pure samples. Imperfect separation will tend to minimise the true differences that exist. In making the division the main clue was the amount of the proper motion; but distance from the apex and apparent magnitude had also to be taken into account in doubtful cases. The giants include

85 F and 90 G stars;

the dwarfs,

114 F and 46 G stars.†

The results are given in Table II. The position of the vertex is in each case just what we should expect from the proportions of mixture of F and G stars with the respective vertices given in

* B. Boss, *Astronomical Journal*, No. 649–50.

† The 17 G stars have been rejected as before irrespective of whether they were dwarfs or giants, but owing to the uncertainty as to how the dwarfs and giants are respectively affected by this rejection, the results are given as they were derived (*i.e.* without multiplication by a factor).

Table I. Thus the separation into giants and dwarfs does not help to explain the discordance of the G vertex.

But the results for the lengths of the axes are very remarkable, the velocities of the dwarfs being much greater than those of the giants in all directions, and especially in the direction of the vertices. This is the more significant because the majority of the G stars (which have the larger motions) are included among the giants.

A similar division has been effected for the K-K₂ stars. In this case, however, the "giant and dwarf" hypothesis requires that practically all the stars of this type in the catalogue should be giants; and our division corresponds to more distant and nearer giants—or, what is nearly equivalent, to the brighter and the fainter sections of the giants. The mean visual magnitude is very nearly the same in the two groups. The greater speed of the nearer stars is again very strongly marked (Table II.). It seems evident from this that the effect cannot be attributed to the existence of two separate divisions—giants and dwarfs,—but represents some continuously varying property.

Three explanations are possible :

- (1) The average velocity decreases with increasing absolute brightness.
- (2) The average velocity decreases with increasing distance from the sun.
- (3) There is a strong correlation between the component velocities in different directions, so that (contrary to the Maxwellian or ellipsoidal law) a large transverse motion is likely to be associated with a large radial motion.

The last explanation must be included because we have used the proper motion as a criterion of distance, virtually assuming that there is no correlation between the components in different directions.

A paper by J. C. Kapteyn and W. S. Adams (*Proceedings of the National Academy of Sciences*, vol. i. p. 14, January 1915) has just come to hand, in which similar results have been found, viz. that for all the spectral classes F, G, K, M the average radial velocities show a regular increase with proper motion. They attribute the result mainly to the third cause mentioned, but find also some positive indications of a dependence of the velocity on absolute magnitude.

6. *Skewness of the Velocity-Distribution.*—In the investigation of the proper motions it has generally been found that there is a skewness of the velocity-distribution which cannot conveniently be represented on the ellipsoidal hypothesis, but is indicated on the two-drift hypothesis by Drift I. having more stars than Drift II. in the ratio of about 3 : 2. This corresponds to a greater number of stars moving towards the vertex 94° , $+12^\circ$, and a smaller number to the opposite vertex; if the solar motion has been correctly eliminated, there will be a compensating smaller

average velocity towards $94^\circ, +12^\circ$, and a greater average velocity to the opposite vertex. The difference (measured in either way) will not be large, and must be between 3 : 2 (the limit when the *motus peculiares* are zero) and 1 : 1 (when the *motus peculiares* are very great). We have examined the stars of types G and K for this effect, the former type because of the discordance of its vertex, and the latter because of the large number of stars available. The stars near the two vertices are examined, because it is for these that the radial components are affected by the skewness that we are considering.

Type.	Distances from Vertices.	No. of Stars moving towards		Average Motion towards	
		$94^\circ, +12^\circ$	$274^\circ, -12^\circ$	$94^\circ, +12^\circ$ km./sec.	$274^\circ, -12^\circ$ km./sec.
G	$0^\circ-40^\circ$	24	15	20.2 *	13.6
G	$0^\circ-50^\circ$	39	26	20.8 *	15.2
K-K ₂	$0^\circ-40^\circ$	42	39	21.3	17.8
K-K ₂	$0^\circ-50^\circ$	69	54	18.2	17.8

From the number of stars moving towards each vertex we should conclude that there is a skewness in the same sense as is shown in the proper motions; but from the average motions (especially in the case of type G) the opposite sign is indicated. The disagreement of the two criteria is evidently due to the use of a value for the solar motion which is not that best fitted to the particular stars considered. For example, taking G ($0^\circ-50^\circ$) and using the speed of the solar motion indicated by these 65 stars (24 instead of 14 km. per sec.), the numbers would be 32 and 33 instead of 39 and 26. It is clear that, owing to uncertainties of the solar motion and to the small number of stars discussed, no conclusion can be drawn except that the skewness (if it exists) must be small.

The frequency-distribution for Type K-K₂ ($0^\circ-50^\circ$) is as follows:—For successive ranges of velocity, < -55 , -55 to -45 , -45 to -35 , . . . , 45 to 55 , > 55 , the numbers are 1, 2, 3, 9, 13, 14, 26, 25, 12, 8, 5, 1, 4. The positive direction of velocity is towards the vertex at $94^\circ, +12^\circ$.

Conclusions.

1. There is a close quantitative agreement between the results derived respectively from the proper motions and the radial velocities in regard to the star-streams. This is shown particularly in:

- (a) Position of the vertex.
- (b) Elongation of the velocity-ellipsoid.
- (c) The diminishing elongation of the ellipsoid for the later spectral types.

* Omitting respectively 2 and 3 velocities > 95 km. per sec.

2. The mean result, depending on 1086 stars (types A-M) is :

Vertex, R.A. $94^{\circ}6$, Dec. $+12^{\circ}5$
 Ratio of axes of velocity-ellipsoid, 0.59

3. There is no evidence of a progressive change with spectral type in the position of the vertex, but there is a large and unexplained deviation in the position of the vertex for Type G.

4. Star-streaming is not shown in the earliest Type A stars (A0), but it is developed fully in the later stars of this type. Stars of Type A0, however, exhibit a strong tendency to move parallel to the plane of the galaxy; no trace of galactic influence is exhibited in the case of other types.

5. The dwarf stars of Types F and G are distinguished by very large average velocities, particularly in the directions of the vertices; but the property does not seem to depend on the existence of two distinct classes. A separation of the K-K2 stars (for which the giant and dwarf division does not arise) into more distant and nearer groups shows a similar difference of average speed.

6. The distribution of velocities appears to be nearly symmetrical with respect to the two vertices; but the material is scarcely sufficient to decide whether there is a skewness similar to that detected in the Boss proper motions.

Cambridge Observatory :
 1915 May 13.

On a Method of solving Spherical Triangles, and performing other Astronomical Computations, by Use of a Simple Table of Squares. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. Computers sometimes find that a method which represents nothing mathematically new, nevertheless has special practical advantages. The following method has been found to facilitate the calculation of distances of seismological observing stations from a given epicentre, and seems to be capable of a wider extension.

Let (λ, δ) be the longitude and latitude of the epicentre, and (l, d) of the observing station. If we put

$$\begin{array}{lll} a = \cos d \cos l & b = \cos d \sin l & c = \sin d \\ \alpha = \cos \delta \cos \lambda & \beta = \cos \delta \sin \lambda & \gamma = \sin \delta \end{array}$$

then the familiar formula of spherical trigonometry

$$\cos \Delta = \sin d \sin \delta + \cos d \cos \delta \cos (l - \lambda)$$

is transformed into the familiar formula of direction-cosines

$$\cos \Delta = a\alpha + b\beta + c\gamma.$$

Now when Δ is small the right-hand side changes very slowly, and the formula only works by retaining many figures. It can, however, be rewritten

$$2 \text{ versin } \Delta = (a - \alpha)^2 + (b - \beta)^2 + (c - \gamma)^2,$$

since

$$a^2 + b^2 + c^2 = \alpha^2 + \beta^2 + \gamma^2 = 1;$$

and in this form it works conveniently. Suppose, for instance, that we have

$$\begin{array}{lll} a = +.595 & b = -.032 & c = +.803 \text{ (Bidston)} \\ a = +.568 & b = -.032 & c = +.822 \text{ (Eskdalemuir)} \end{array}$$

then even if we retain all the figures Crelle gives (which is otherwise not legitimate) we get

$$a\alpha + b\beta + c\gamma = .999050,$$

which assigns Δ as $2^\circ.5$ or thereabouts. But a very modest table of squares gives

$$(a - \alpha)^2 + (b - \beta)^2 + (c - \gamma)^2 = .00109;$$

and from a table of versines (or, better still, a special table of double versines *) we find $\Delta = 1^\circ.9$, showing that the $\cos \Delta$ formula gives not merely an indefinite but an erroneous value, the fact being that the extra figures retained in .99905 are misleading, unless we carry a, b, c , etc., to more places of decimals.

2. Having thus started this formula for a special case, one was led to enquire into the limits of its sphere of usefulness; and it was soon found that there was no need of restriction. One great advantage of the formula is the disappearance of signs, the three quantities to be added being all squares. Signs must be regarded in *forming* the differences, of course, but it is not necessary to record the sign of the *results*—merely to note whether the numerical sum or the numerical difference is to be taken. This is in the first instance a convenience, and in the second a safeguard against mistakes. But there is an even more important side to the matter, in the total avoidance of cases where a small result is rendered inaccurate by reason of being the difference between two much larger quantities. A sum of three squares can only be small by all three being small; and if two of them are much larger than the third, the third cannot appear unexpectedly important by the cancelling of the two large ones.

Finally, it is worth remarking that a table of squares and a table of double versines (or, in default of these, of simple cosines properly extended near zero) are much handier than Crelle's tables.

* A table of double versines to four figures is appended to this paper.

If multiplication tables, or a multiplying machine, are at hand, the formula

$$\cos \Delta = a\alpha + b\beta + c\gamma$$

may be more convenient and quicker. But there are occasions when these are not easily available (let us say in the field), but when a copy of Bottomley's Tables can be carried in the pocket.

3. When Δ is greater than 90° , we have the alternative formula

$$2(1 + \cos \Delta) = (\alpha + a)^2 + (b + \beta)^2 + (c + \gamma)^2;$$

in fact we use the antipodes of one of the points, and find $180^\circ - \Delta$.

4. In transforming co-ordinates completely we must find at least two, and the above formula seems to supply only one, viz. the distance EO or Δ , separating the observing station O from the epicentre E. To find the angle PEO which this distance makes with the meridian EP, we have

$$\sin \Delta \sin \text{PEO} = \cos d \sin (l - \lambda),$$

which is probably the best way of finding PEO. But we can, if desired, throw this also into the form of three squares; for

$$2 \cos d \sin (l - \lambda) = 2b \cos \lambda - 2a \sin \lambda.$$

Thus

$$(2 + 2 \sin \Delta \sin \text{PEO}) = (a - \sin \lambda)^2 + (b + \cos \lambda)^2 + c^2;$$

and similarly we have

$$(2 + 2 \sin \Delta \cos \text{PEO}) = (a - \sin \delta \cos \lambda)^2 + (b - \sin \delta \sin \lambda)^2 + (c + \cos \delta)^2.$$

For a given (λ, δ) of the epicentre, the quantities to be subtracted from (a, b, c) are constants easily calculated. This uniform process in fact obtains the double versines of the distances of any station from three fixed points on the sphere, viz., the epicentre itself, and two points 90° distant from it, one of which is in its meridian. To get the angle PEO from these formulæ we must divide $2 \sin \Delta \sin \text{PEO}$ by $2 \sin \Delta \cos \text{PEO}$; or alternatively, either of them by $2 \sin \Delta$. It will be seen that the proper quadrant is assigned by the *magnitudes* and not the signs of the sums of squares. These are always positive; but the question is whether they are greater or less than 2.

5. This method was adopted in the first instance, as already stated, for calculations with 3 figures. But it seems to be capable of an important extension—say to 8 figures. Our present logarithm tables are to 7 figures, and in some cases fall just short of the desired accuracy. Tables to 8 figures have been made, but are cumbersome and sometimes costly. There is no difficulty in giving 8 figures, or even 18, in the *logarithm*; but there are either troublesome interpolations in using an *argument* to 8 figures, or the tables must be made very bulky.

6. Now with a table of *complete* squares from 1000 to 9999

(such as Barlow's Tables, or a volume even more compact giving squares only), the complete square of any 8-figure number can quickly be found as follows. Represent it by $a + 10^{-4}b$, where a denotes the first 4 figures and b the last 4.

Then

$$(a + 10^{-4}b)^2 = a^2 + 10^{-4}2ab + 10^{-8}b^2.$$

But

$$2ab = a^2 + b^2 - (a - b)^2.$$

Hence

$$\begin{aligned}(a + 10^{-4}b)^2 &= a^2 + 10^{-4}a^2 \\ &\quad + 10^{-4}b^2 + 10^{-8}b^2 \\ &\quad - 10^{-4}(a - b)^2.\end{aligned}$$

7. An example will make the method clear. To find the square of 48723019 :

Square of 4872	= 23736384	(1)
Repeat 4 places to right	= 23736384	(2)
Square of 3019	= 9114361	(3)
Repeat 4 places to right	= 9114361	(4)
Subtract square of 1853	= - 3433609	(5)
Square of 48723019	= 2373932580474361	

There need be no hesitation in placing the figures if we note that line (2) is line (1) shifted 4 places; that lines (3) and (5) must *end* in the same place as line (2); and that line (4) hangs over 4 places beyond the end. The negative sign for line (5) is a slight disadvantage, but, coming at the bottom, it is easy to fix attention on it. The whole of the figures have been given to show the method, but in much work we could neglect all but the first 8 if we are working throughout to 8 places.

8. Hence any formula of spherical trigonometry which can be thrown into the form

$$\cos \Delta = Aa + Bb + Cc,$$

and therefore into the form

$$2 - 2 \cos \Delta = (A - a)^2 + (B - b)^2 + (C - c)^2,$$

can be solved to 8 places with Barlow's Tables, so far as finding $\cos \Delta$ to 8 figures. To find Δ we must of course have 8-figure trigonometrical tables; but for many purposes we might be content to stop at $\cos \Delta$. Thus if, instead of, or in addition to, tabulating α and δ in our catalogues, we tabulate

$$l = \cos \alpha \cos \delta \quad m = \sin \alpha \cos \delta \quad n = \sin \delta,$$

then we can convert the (l, m, n) of all the stars for one epoch into those for any other epoch very exactly by formulæ of this kind,

needing only complete squares of numbers from 1000 to 9999. To get the R.A. at the latter epoch we must use the formula

$$\tan RA = n/l,$$

and as yet I have not been able to see how to do this division very easily. If we could easily find the reciprocal of l , the division would become a multiplication. Barlow's tables give reciprocals of 4 figures to 7 places only. If they were extended to 8 figures, then to find the reciprocal of an 8-figure number $a + 10^{-4}b$, put R for the reciprocal of a , given by the tables, and let $(R - x)$ be that of $(a + 10^{-4}b)$, so that

$$(R - x)(a + 10^{-4}b) = 1 = Ra.$$

Then

$$x = 10^{-4}b \cdot R^2 \text{ approximately,}$$

which is in the form of a multiplication; but the process is becoming lengthy, and would probably only be useful when logarithm tables are not at hand.

9. For converting (l, m, n) into a similar system referred to the galaxy, the process would probably be found far better than the usual logarithmic processes.

10. The fact that a multiplication can be performed by using squares has long been recognised by the formal tabulation of "quarter squares," the equivalence being

$$Aa = (A + a)^2/4 - (A - a)^2/4.$$

The method given above uses the relation

$$2Aa = A^2 + a^2 - (A - a)^2,$$

which, for a single term, is a step in the wrong direction, substituting three squares for two. It is the combination ($A^2 + B^2 + C^2 = 1$ and $a^2 + b^2 + c^2 = 1$) which in the case of three terms, $Aa + Bb + Cc$, gives the method its advantages. Now at first sight this might seem an exceptional case not likely to occur in other connections. But the recurrence is not infrequent.

11. Notice, first, that there is no need for any special relation among the A^2 , B^2 , etc., if we take A , B , etc., to be a set of constants which are to multiply a set of variables a , b , etc. The sum ΣA^2 will be a constant in any case, and though it may not be so simple as unity, it can be added (or subtracted) without much difficulty when the whole final process is one of addition (or subtraction).

12. Secondly, when the variables a , b , etc., are a set of sines and cosines, or products of these, instances when they add up to unity are not infrequent. Roughly speaking, they are the cases which we have been accustomed to see laboriously "adapted to logarithmic computation" by the use of subsidiary angles.

13. A good instance is furnished by the formulæ for star corrections. We have Bessel's formulæ $Aa + Bb + Cc + Dd$ and $Aa' + Bb' + Cc' + Dd'$; and then we have the "adapta-

tions" $f + \frac{1}{15}g \cos (G + a) \tan \delta + \frac{1}{15}h \sin (H + a) \sec \delta$ and $i \cos \delta + g \cos (G + a) + h \cos (H + a) \sin \delta$. But these can be easily calculated as sums of squares. Take the latter. We can write

$$\begin{aligned} 2i \cos \delta &= (i + \cos \delta)^2 - i^2 - \cos^2 \delta \\ 2h \cos H \cos a \sin \delta &= (h \cos H + \cos a \sin \delta)^2 - h^2 \cos^2 H - \cos^2 a \sin^2 \delta \\ - 2h \sin H \sin a \sin \delta &= (h \sin H - \sin a \sin \delta)^2 - h^2 \sin^2 H - \sin^2 a \sin^2 \delta \\ 2g \cos G \cos a &= (g \cos G + \cos a)^2 - g^2 \cos^2 G - \cos^2 a \\ - 2g \sin G \sin a &= (g \sin G - \sin a)^2 - g^2 \sin^2 G - \sin^2 a \end{aligned}$$

It is clear that on the right-hand side we have, after the terms in brackets, which represent the main part of the computation :

First, a set of terms which add up to $-(i^2 + h^2 + g^2)$, not unity, but a constant which is the same for all stars on that day and can be given as a "day number."

Second, a series of terms which add up to -2 for all stars, being squares of sines and cosines and products, and can thus be included with the former as $-(2 + i^2 + h^2 + g^2)$.

14. The process of computation thus consists of

(1) The writing down of $\cos a$, $-\sin a$, and the products of these by $\sin \delta$.

(2) The addition of these to five "day numbers": i , $h \cos H$, $h \sin H$, $g \cos G$, $g \sin G$.

(3) The formation of five corresponding squares in which signs may be neglected.

(4) The subtraction of the "day number" $(2 + i^2 + h^2 + g^2)$.

15. No special tables are required: 4-figure tables giving natural sines and cosines and squares (such as Bottomley's) give all that is necessary; and this is in itself an advantage, since much labour has already been spent on the construction of special tables.* Moreover, the star correction in R.A. can be thrown into the same form, using almost precisely the same star constants, if we write (putting $F = 15f$)

$$\begin{aligned} 2 \times 15 \cos \delta \Delta a &= 2F \cos \delta + 2h \sin (H + a) + 2g \sin (G + a) \sin \delta \\ &= (F + \cos \delta)^2 + (h \sin H + \cos a)^2 + (h \cos H + \sin a)^2 \\ &\quad + (g \sin G + \cos a \sin \delta)^2 \\ &\quad + (g \cos G + \sin a \sin \delta)^2 \\ &\quad - (2 + F^2 + h^2 + g^2). \end{aligned}$$

The only novelties here are the substitution of $-(2 + F^2 + h^2 + g^2)$ for $-(2 + i^2 + h^2 + g^2)$ and the final division by $15 \cos \delta$, which is reasonably reserved till the end, since it represents conversion from arc on the sphere into a rather conventional unit.

* The present writer pleads guilty to the accusation of sharing in this superfluity.

16. But there are two possible improvements in detail which call for consideration.

First, it will be just as well to get rid of the factor 2 if we conveniently can. Such a factor is apt to cause mistakes. Now we can get rid of it easily in either of two ways—by dividing the “day numbers” by 2, or the “star numbers,” *i.e.* we may write

$$Aa = (\frac{1}{2}A + a)^2 - \frac{1}{4}A^2 - a^2$$

or

$$Aa = (A + \frac{1}{2}a)^2 - A^2 - \frac{1}{4}a^2.$$

The choice is guided by the second consideration, which is as follows: When we are *multiplying* A and a together, it does not matter if they are of widely different orders of magnitude; but when we are *adding* them (to form a sum which is to be squared), it is convenient to have them of the same order. In the case of distances between one point on the sphere and another, where all the quantities are direction-cosines, this condition is secured automatically. In the case of star corrections, A and B , or $h \sin H$, and $h \cos H$, represent aberration and are of the order 20"; C (replaced above by f and g , etc.) represents precession, and within one year is of the order 50". It is not convenient to add sines and cosines, all less than unity, to numbers like 20 and 50. We must divide the former or multiply the latter by some simple factor.

17. Combining this desideratum with the former, we arrive at the suggestions:—

Divide the day numbers by 10 (second consideration), which reduces the largest of them (C) to the order 5 instead of 50.

Multiply the star numbers by 10 (second consideration) and divide by 2 (first consideration), which raises them to the order 5, on a par with the day numbers.

The star correction is thus unaltered, the factor 2 being got rid of. It will suffice to exhibit a single term, such as

$$i \cos \delta = (i/10 + 5 \cos \delta)^2 - i^2/100 - 25 \cos^2 \delta.$$

18. The day numbers now take the form

$$\frac{1}{10}f \quad \frac{1}{10}h \cos H \quad \frac{1}{10}h \sin H \quad \frac{1}{10}g \cos G \quad \frac{1}{10}g \sin G \quad \frac{1}{10}i$$

to which are to be added, to form $15 \cos \delta \cdot \Delta a$

$$5 \cos \delta \quad 5 \sin a \quad 5 \cos a \quad 5 \sin a \sin \delta \quad 5 \cos a \sin \delta \quad (\text{omit})$$

and to form $\Delta \delta$, we have the numbers

$$(\text{omit}) \quad 5 \cos a \sin \delta \quad -5 \sin a \sin \delta \quad 5 \cos a \quad -5 \sin a \quad 5 \cos \delta$$

It will be seen how readily these may be written down, one row being the other written backwards, with two changes of sign and attention to the omissions. There are two additional subtractions, *viz.*, $\{50 + (15f)^2 + h^2 + g^2/100\}$ and $\{50 + (i^2 + h^2 + g^2)/100\}$ and finally the division of $15 \cos \delta \cdot \Delta a$ by $15 \cos \delta$.

19. An example will show the amount of work involved, when once the day numbers are given. For the multiplication by $\sin \delta$ and the division by $15 \cos \delta$ it seems difficult to avoid the use of Crelle, unless the operator will face a little simple arithmetic. As an example take the place of δ Ursæ Minoris on 1899 Dec. 7. The day numbers, given in the above form, are

$$+5.81 \quad +1.98 \quad +0.46 \quad +2.53 \quad +0.19 \quad +0.20$$

The mean place of δ Ursæ is $\alpha = 271^\circ 13'$, $\delta = 86^\circ 37'$.

The R.A. star numbers are therefore

$$+0.30 \quad -5.00 \quad +0.11 \quad -4.99 \quad +0.11 \quad \dots$$

and the Decl. star numbers are these written backwards, viz.,

$$\dots \quad +0.11 \quad +4.99 \quad +0.11 \quad +5.00 \quad +0.30$$

Hence we have

$$15 \cos \delta \cdot \Delta \alpha = (6.11)^2 + (3.02)^2 + (0.57)^2 + (2.46)^2 + (0.30)^2 - 94.32 \text{ (day number)}$$

$$\Delta \delta = (2.09)^2 + (5.45)^2 + (2.64)^2 + (5.19)^2 + (0.50)^2 - 60.60 \text{ (day number)}$$

Whence

$$\Delta \alpha = -41.41/15 \cos \delta = -46^s.80$$

$$\Delta \delta = +7''.64.$$

In comparing with the *N.A.* we must remember that the day constants are taken out for midnight, whereas the *N.A.* apparent place is calculated for upper transit, 11 hours earlier. Further, the *N.A.* place is corrected for the short period terms. Making these allowances, the accordance is satisfactory.

20. For θ' Microscopii on the same day, $\alpha = 318^\circ 35'$, $\delta = -41^\circ 14'$, the numbers are

Day	+5.81	+1.98	+0.46	+2.53	+0.19	+0.20	Day
R.A.	+3.76	-3.31	+3.75	+2.18	-2.47	...	94.32
Dec.	...	-2.47	-2.18	+3.75	+3.31	+3.76	60.60

$$15 \cos \delta \Delta \alpha = (9.57)^2 + (1.33)^2 + (4.21)^2 + (4.71)^2 + (2.28)^2 - 94.32 = +44''.10$$

$$\Delta \delta = (0.49)^2 + (1.72)^2 + (6.28)^2 + (3.50)^2 + (3.96)^2 - 60.60 = +9''.88$$

Hence $\Delta \alpha = +3^s.91$, $\Delta \delta = +9''.88$. The *N.A.* values are $+3^s.92$ and $+9''.9$.

21. It will be seen that three figures only are required. The star numbers and day numbers are all less than 5.00, so that the sums never exceed 9.99. The squares are thus taken out from Bottomley's double-page table *without interpolation*—a great advantage. The value of Crelle's tables for the multiplications by $\sin \delta$ and (for R.A.) by $\sec \delta / 15$ must be admitted; but the work is not serious.

22. The process of adding two numbers and then finding the square is of course admirably adapted for a slide-rule; and it

would not be difficult to devise a machine which would give all five squares at sight.

Summary.

If the position of a fixed point (such as an earthquake epicentre) be denoted by the direction-cosines (A, B, C), and that of a variable point (such as an observing station) by (a, b, c), the advantages of the formula

$$2 \text{ versin } \Delta = (A - a)^2 + (B - b)^2 + (C - c)^2$$

are pointed out.

In the above case, A, a , etc., are often only required to 3 figures, when the table of squares can be on a double page.

The method, however, can be used in work of the greatest precision (say 8 figures) without much tabulation, since by the device shown in § 6 squares of 8 figures can quickly be obtained from a table of *complete* squares from 1000 to 9999.

Hence the method is a useful general method for the transformation of co-ordinates.

The method can also be adapted for use in calculating star corrections, and promises a considerable saving of labour in that work.

TABLE OF 2 VERSIN Δ .

I. In Units of the Sixth Decimal Place.

°	'0.	'1.	'2.	'3.	'4.	'5.	'6.	'7.	'8.	'9.	°
0	0	3	12	27	49	76	110	149	195	247	0
1	305	369	439	515	597	685	780	880	987	1100	1
2	1218	1343	1474	1611	1754	1904	2059	2220	2388	2561	2
3	2741	2927	3118	3316	3520	3730	3947	4169	4397	4631	3
4	4872	5118	5370	5630	5894	6165	6442	6725	7014	7309	4

II. In Ordinary Units.

5	'0076	'0079	'0082	'0086	'0089	'0092	'0096	'0099	'0102	'0106	5
6	'0110	'0113	'0117	'0121	'0125	'0129	'0133	'0137	'0141	'0145	6
7	'0149	'0153	'0158	'0162	'0167	'0171	'0176	'0180	'0185	'0190	7
8	'0195	'0200	'0204	'0209	'0215	'0220	'0225	'0230	'0235	'0241	8
9	'0246	'0252	'0257	'0263	'0269	'0274	'0280	'0286	'0292	'0298	9
10	'0304	'0310	'0316	'0322	'0329	'0335	'0341	'0348	'0354	'0361	10
11	'0368	'0374	'0381	'0388	'0395	'0401	'0409	'0415	'0423	'0430	11
12	'0437	'0444	'0452	'0459	'0467	'0474	'0482	'0489	'0497	'0505	12
13	'0513	'0521	'0528	'0536	'0544	'0553	'0561	'0569	'0577	'0586	13
14	'0594	'0603	'0611	'0620	'0628	'0637	'0646	'0655	'0664	'0672	14

TABLE OF 2 VERSIN Δ —*continued.*

	'0.	'1.	'2.	'3.	'4.	'5.	'6.	'7.	'8.	'9.	
15	'0681	'0691	'0700	'0709	'0718	'0727	'0737	'0746	'0756	'0765	15
16	'0775	'0784	'0794	'0804	'0814	'0824	'0834	'0844	'0854	'0864	16
17	'0874	'0884	'0894	'0905	'0915	'0926	'0936	'0947	'0957	'0968	17
18	'0979	'0990	'1001	'1011	'1022	'1034	'1045	'1056	'1067	'1078	18
19	'1090	'1101	'1112	'1124	'1136	'1147	'1159	'1171	'1182	'1194	19
20	'1206	'1218	'1230	'1242	'1254	'1267	'1279	'1291	'1303	'1316	20
21	'1328	'1341	'1354	'1366	'1379	'1392	'1404	'1417	'1430	'1443	21
22	'1456	'1469	'1483	'1496	'1509	'1522	'1536	'1549	'1563	'1576	22
23	'1590	'1604	'1617	'1631	'1645	'1659	'1673	'1687	'1701	'1715	23
24	'1729	'1743	'1758	'1772	'1786	'1801	'1815	'1830	'1844	'1859	24
25	'1874	'1889	'1903	'1918	'1933	'1948	'1963	'1978	'1994	'2009	25
26	'2024	'2039	'2055	'2070	'2086	'2101	'2117	'2133	'2148	'2164	26
27	'2180	'2196	'2212	'2228	'2244	'2260	'2276	'2292	'2308	'2325	27
28	'2341	'2358	'2374	'2390	'2407	'2424	'2440	'2457	'2474	'2491	28
29	'2508	'2525	'2542	'2559	'2576	'2593	'2610	'2627	'2645	'2662	29
30	'2679	'2697	'2715	'2732	'2750	'2768	'2785	'2803	'2821	'2839	30
31	'2857	'2875	'2893	'2911	'2929	'2947	'2965	'2984	'3002	'3020	31
32	'3039	'3058	'3076	'3095	'3114	'3132	'3151	'3170	'3189	'3208	32
33	'3227	'3246	'3265	'3284	'3303	'3322	'3342	'3361	'3380	'3400	33
34	'3419	'3439	'3458	'3478	'3498	'3517	'3537	'3557	'3577	'3597	34
35	'3617	'3637	'3657	'3677	'3697	'3717	'3738	'3758	'3779	'3799	35
36	'3820	'3840	'3861	'3881	'3902	'3923	'3944	'3964	'3985	'4006	36
37	'4027	'4048	'4069	'4091	'4112	'4133	'4154	'4176	'4197	'4218	37
38	'4240	'4261	'4283	'4304	'4326	'4348	'4370	'4391	'4413	'4435	38
39	'4457	'4479	'4501	'4523	'4545	'4567	'4590	'4612	'4634	'4657	39
40	'4679	'4701	'4724	'4747	'4769	'4792	'4815	'4837	'4860	'4883	40
41	'4906	'4929	'4952	'4975	'4998	'5021	'5044	'5067	'5090	'5114	41
42	'5137	'5160	'5184	'5207	'5230	'5254	'5278	'5302	'5325	'5349	42
43	'5373	'5397	'5420	'5444	'5468	'5493	'5517	'5541	'5565	'5589	43
44	'5613	'5637	'5662	'5686	'5710	'5735	'5760	'5784	'5809	'5833	44
45	'5858	'5882	'5907	'5932	'5957	'5982	'6007	'6032	'6057	'6082	45
46	'6107	'6132	'6157	'6182	'6208	'6233	'6258	'6284	'6309	'6335	46
47	'6360	'6386	'6411	'6436	'6462	'6488	'6514	'6540	'6566	'6591	47
48	'6617	'6643	'6669	'6695	'6721	'6748	'6774	'6800	'6826	'6852	48
49	'6879	'6905	'6932	'6958	'6984	'7011	'7038	'7064	'7091	'7118	49
50	'7144	'7171	'7198	'7225	'7252	'7278	'7306	'7332	'7360	'7386	50
51	'7414	'7441	'7468	'7495	'7522	'7550	'7577	'7604	'7632	'7659	51
52	'7687	'7714	'7742	'7769	'7797	'7825	'7852	'7880	'7908	'7936	52
53	'7964	'7992	'8019	'8047	'8075	'8104	'8132	'8160	'8188	'8216	53
54	'8244	'8273	'8301	'8329	'8357	'8386	'8414	'8443	'8471	'8500	54

TABLE OF 2 VERSIN Δ —*continued.*

	°.	'1.	'2.	'3.	'4.	'5.	'6.	'7.	'8.	'9.	°
55	'8528	'8556	'8586	'8614	'8643	'8672	'8701	'8729	'8758	'8787	55
56	'8816	'8845	'8874	'8903	'8932	'8961	'8990	'9020	'9049	'9078	56
57	'9107	'9137	'9166	'9195	'9225	'9254	'9283	'9313	'9342	'9372	57
58	'9402	'9431	'9461	'9491	'9520	'9550	'9580	'9610	'9639	'9669	58
59	'9699	'9729	'9759	'9789	'9819	'9849	'9879	'9909	'9939	'9969	59
60	1'000	1'003	1'006	1'009	1'012	1'015	1'018	1'021	1'024	1'027	60
61	1'030	1'033	1'036	1'040	1'043	1'046	1'049	1'052	1'055	1'058	61
62	1'061	1'064	1'067	1'070	1'073	1'077	1'080	1'083	1'086	1'089	62
63	1'092	1'095	1'098	1'101	1'104	1'108	1'111	1'114	1'117	1'120	63
64	1'123	1'126	1'130	1'133	1'136	1'139	1'142	1'145	1'148	1'152	64
65	1'155	1'158	1'161	1'164	1'167	1'171	1'174	1'177	1'180	1'183	65
66	1'187	1'190	1'193	1'196	1'199	1'203	1'206	1'209	1'212	1'215	66
67	1'219	1'222	1'225	1'228	1'231	1'235	1'238	1'241	1'244	1'248	67
68	1'251	1'254	1'257	1'261	1'264	1'267	1'270	1'273	1'277	1'280	68
69	1'283	1'287	1'290	1'293	1'296	1'300	1'303	1'306	1'309	1'313	69
70	1'316	1'319	1'323	1'326	1'329	1'332	1'336	1'339	1'342	1'346	70
71	1'349	1'352	1'355	1'359	1'362	1'365	1'369	1'372	1'375	1'379	71
72	1'382	1'385	1'389	1'392	1'395	1'399	1'402	1'405	1'409	1'412	72
73	1'415	1'419	1'422	1'425	1'429	1'432	1'435	1'439	1'442	1'445	73
74	1'449	1'452	1'455	1'459	1'462	1'466	1'469	1'472	1'476	1'479	74
75	1'482	1'486	1'489	1'492	1'496	1'499	1'503	1'506	1'509	1'513	75
76	1'516	1'520	1'523	1'526	1'530	1'533	1'537	1'540	1'543	1'547	76
77	1'550	1'553	1'557	1'560	1'564	1'567	1'571	1'574	1'577	1'581	77
78	1'584	1'588	1'591	1'594	1'598	1'601	1'605	1'608	1'612	1'615	78
79	1'618	1'622	1'625	1'629	1'632	1'636	1'639	1'642	1'646	1'649	79
80	1'653	1'656	1'660	1'663	1'666	1'670	1'673	1'677	1'680	1'684	80
81	1'687	1'691	1'694	1'697	1'701	1'704	1'708	1'711	1'715	1'718	81
82	1'722	1'725	1'729	1'732	1'735	1'739	1'742	1'746	1'749	1'753	82
83	1'756	1'760	1'763	1'767	1'770	1'774	1'777	1'781	1'784	1'787	83
84	1'791	1'794	1'798	1'801	1'805	1'808	1'812	1'815	1'819	1'822	84
85	1'826	1'829	1'833	1'836	1'840	1'843	1'847	1'850	1'854	1'857	85
86	1'860	1'864	1'867	1'871	1'874	1'878	1'881	1'885	1'888	1'892	86
87	1'895	1'899	1'902	1'906	1'909	1'913	1'916	1'920	1'923	1'927	87
88	1'930	1'934	1'937	1'941	1'944	1'948	1'951	1'955	1'958	1'962	88
89	1'965	1'969	1'972	1'976	1'979	1'983	1'986	1'990	1'993	1'997	89
90	2'000	2'003	2'007	2'010	2'014	2'017	2'021	2'024	2'028	2'031	90
91	2'035	2'038	2'042	2'045	2'049	2'052	2'056	2'059	2'063	2'066	91
92	2'070	2'073	2'077	2'080	2'084	2'087	2'091	2'094	2'098	2'101	92
93	2'105	2'108	2'112	2'115	2'119	2'122	2'126	2'129	2'133	2'136	93
94	2'140	2'143	2'146	2'150	2'153	2'157	2'160	2'164	2'167	2'171	94

TABLE OF 2 VERSIN Δ —*continued*.

	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	
95	2'174	2'178	2'181	2'185	2'188	2'192	2'195	2'199	2'202	2'206	95
96	2'209	2'213	2'216	2'219	2'223	2'226	2'230	2'233	2'237	2'240	96
97	2'244	2'247	2'251	2'254	2'258	2'261	2'265	2'268	2'271	2'275	97
98	2'278	2'282	2'285	2'289	2'292	2'296	2'299	2'303	2'306	2'309	98
99	2'313	2'316	2'320	2'323	2'327	2'330	2'334	2'337	2'340	2'344	99
100	2'347	2'351	2'354	2'358	2'361	2'364	2'368	2'371	2'375	2'378	100
101	2'382	2'385	2'388	2'392	2'395	2'399	2'402	2'406	2'409	2'412	101
102	2'416	2'419	2'423	2'426	2'429	2'433	2'436	2'440	2'443	2'447	102
103	2'450	2'453	2'457	2'460	2'463	2'467	2'470	2'474	2'477	2'480	103
104	2'484	2'487	2'491	2'494	2'497	2'501	2'504	2'508	2'511	2'514	104
105	2'518	2'521	2'524	2'528	2'531	2'534	2'538	2'541	2'545	2'548	105
106	2'551	2'555	2'558	2'561	2'565	2'568	2'571	2'575	2'578	2'581	106
107	2'585	2'588	2'591	2'595	2'598	2'601	2'605	2'608	2'611	2'615	107
108	2'618	2'621	2'625	2'628	2'631	2'635	2'638	2'641	2'645	2'648	108
109	2'651	2'654	2'658	2'661	2'664	2'668	2'671	2'674	2'677	2'681	109
110	2'684	2'687	2'691	2'694	2'697	2'700	2'704	2'707	2'710	2'713	110
111	2'717	2'720	2'723	2'727	2'730	2'733	2'736	2'739	2'743	2'746	111
112	2'749	2'752	2'756	2'759	2'762	2'765	2'769	2'772	2'775	2'778	112
113	2'781	2'785	2'788	2'791	2'794	2'797	2'801	2'804	2'807	2'810	113
114	2'813	2'817	2'820	2'823	2'826	2'829	2'833	2'836	2'839	2'842	114
115	2'845	2'848	2'852	2'855	2'858	2'861	2'864	2'867	2'870	2'874	115
116	2'877	2'880	2'883	2'886	2'889	2'892	2'896	2'899	2'902	2'905	116
117	2'908	2'911	2'914	2'917	2'920	2'923	2'927	2'930	2'933	2'936	117
118	2'939	2'942	2'945	2'948	2'951	2'954	2'957	2'960	2'964	2'967	118
119	2'970	2'973	2'976	2'979	2'982	2'985	2'988	2'991	2'994	2'997	119

The table should not be used near 180° . It is preferable to use the formula $(a + A)^2 + (b + B)^2 + (c + C)^2$ instead of $(a - A)^2 + (b - B)^2 + (c - C)^2$, as explained in § 3. But, of course, values for angles greater than 90° can be obtained by subtracting from 4 an angle less than 90° .

A Preliminary Discussion of Three Years' Observations made with the Cookson Floating Zenith-Telescope at the Royal Observatory, Greenwich. By H. S. Jones, M.A., B.Sc. (Plate 17.)

(Communicated by the Astronomer Royal.)

The floating zenith-telescope designed by the late Mr. Bryan Cookson, and after his death presented to the Cambridge University Observatory by Mrs. Cookson, was lent, in 1911, to the Royal Observatory, Greenwich, for a period of seven years, for a series of observations for the determination of the aberration constant. A description of the instrument was given in *Memoirs, R.A.S.*, vol. lx. pt. ii., and the method of observing, and of measuring the plates, and the observing programme adopted at Greenwich were described by Prof. Eddington in *M.N.*, *R.A.S.*, vol. lxxiii. p. 605. As half the period to be covered by the observations is now completed, it seemed desirable to make a preliminary discussion of these observations, in order to obtain some information as to their accuracy and value, the more so as the value of the aberration constant to be derived from the whole series of observations should not greatly differ from that furnished by the first half of them. The present communication consists of a brief account of the results derived from the observations made since their commencement in September 1911 until the end of the year 1914. Excluding rejected plates, the discussion is based upon the measures of 479 plates, of which number 29, which show only two pairs of star-trails, have been given half-weight.

From the whole series of observations reduced to the epoch 1912.0,* preliminary values of the mean zenith distance of each pair of stars in the observing programme were first derived, and the residuals obtained by subtracting these from the observed values taken, and the mean residual for each plate used in the subsequent discussion. The observing programme contains 16 groups, observed in a cyclical manner, the year being divided into 10 periods. Thirty-three such periods are embraced by the plates here discussed. The plates of a given group (the p th), which are observed during the q th period, give an equation of condition of the form :—

$$z_p + t_q + a_{pq}K = \text{mean residual}$$

where z_p is the mean correction to the assumed star places for the p th group, t_q is the latitude variation during the q th period, a_{pq} is the mean aberration factor for this group and period, and $20''.47K$ is the correction to the assumed value, $20''.47$, of the aberration constant. The weight of the equation is the weighted number of plates upon which it is based, plates with two pairs of trails upon them having weight $\frac{1}{2}$, and those with more than two having weight 1.

* The small nutation terms were included in the star corrections in making this reduction.

In order to get some idea as to the dependence of the deduced value of the aberration constant upon the parallax of the stars used, it was assumed that the mean parallax of the stars in each group is the same, and of amount π . The mean magnitudes and proper motions of the stars in the various groups not being greatly different, is sufficient justification for this approximation.

If β_{pq} is the parallax factor, the equation of condition is then

$$z_p + t_q + \alpha_{pq}K + \beta_{pq}\pi = \text{mean residual.}$$

From these equations, normal equations for the 16 z 's, 33 t 's, for K and for π were constructed. Each z normal equation involves several t 's (11 at most), and each t normal equation involves not more than 4 z 's. The K and π equations involve all the z 's and t 's.

These equations were solved by successive approximation. The values of the z 's, being small, were at first assumed zero, and values of the t 's obtained in terms of π and K from the t normal equations. Substitution of these values in the z normal equations gave values for the z 's, and, using these, corrected values of the t 's were then obtained. The values of the z 's and t 's, thus expressed in terms of K and of π , were substituted in the normal equations for these quantities.

The values of the mean latitude variation in the various periods thus derived are given in the following table, from which can be seen the extent to which they are dependent upon the assumed value of the aberration constant and upon the parallax of the stars used:—

TABLE I.

$t_1 = -\cdot 121 + 6\cdot 0K + 74\pi$	$t_{18} = -\cdot 067 + 2\cdot 8K + 98\pi$
$t_2 = -\cdot 130 + 8\cdot 1 + 39$	$t_{19} = +\cdot 121 - 2\cdot 3 + 93$
$t_3 = -\cdot 156 + 12\cdot 1 + 05$	$t_{20} = +\cdot 121 + 1\cdot 7 + 79$
$t_4 = -\cdot 041 + 1\cdot 7 + 04$	$t_{21} = +\cdot 075 + 4\cdot 7 + 74$
$t_5 = -\cdot 042 - 4\cdot 5 + 37$	$t_{22} = +\cdot 091 + 8\cdot 7 + 39$
$t_6 = +\cdot 180 - 4\cdot 7 + 74$	$t_{23} = +\cdot 212 + 3\cdot 2 + 04$
$t_7 = +\cdot 264 - 0\cdot 9 + 90$	$t_{24} = +\cdot 155 - 4\cdot 4 + 04$
$t_8 = +\cdot 161 - 0\cdot 3 + 92$	$t_{25} = -\cdot 001 - 5\cdot 2 + 40$
$t_9 = +\cdot 161 - 1\cdot 0 + 94$	$t_{26} = -\cdot 052 - 2\cdot 7 + 78$
$t_{10} = +\cdot 207 - 2\cdot 2 + 79$	$t_{27} = -\cdot 064 - 2\cdot 1 + 91$
$t_{11} = +\cdot 100 + 6\cdot 2 + 75$	$t_{28} = -\cdot 116 + 2\cdot 1 + 96$
$t_{12} = +\cdot 103 + 4\cdot 8 + 42$	$t_{29} = -\cdot 133 + 0\cdot 3 + 94$
$t_{13} = -\cdot 039 + 7\cdot 4 + 08$	$t_{30} = -\cdot 199 + 4\cdot 1 + 78$
$t_{14} = -\cdot 085 + 4\cdot 1 + 02$	$t_{31} = -\cdot 081 + 75$
$t_{15} = -\cdot 150 + 1\cdot 8 + 40$	$t_{32} = -\cdot 005 + 4\cdot 8 + 41$
$t_{16} = -\cdot 107 - 2\cdot 5 + 75$	$t_{33} = +\cdot 140 + 4\cdot 1 + 04$
$t_{17} = -\cdot 085 - 2\cdot 8 + 89$	

If the value of the aberration constant is in error by as much as $0''.02$, then $K = 0.001$, and the maximum effect on any of the t 's is only $0''.01$. The mean parallax of the stars affects the latitude variation almost to the full extent of its value: for the stars in the Cookson programme, the mean values of the parallax, taken from Kapteyn's tables in *Groningen Publications*, No. 8, from a knowledge of their spectral types, magnitudes, and proper motions, is $0''.011$, so that the influence of the parallax on the latitude variation is not very considerable.

The normal equation for K , after substituting for the z 's and t 's, reduced to

$$25,729K + 16.48\pi + 3.2 = 0,$$

or, if $20.47K = \kappa$, the correction to the assumed value of the aberration, then

$$1252\kappa + 16.48\pi + 3.2 = 0.$$

π is certainly less than $+0''.02$, but if we take this latter value, the constant term is only changed from 3.2 to 3.5 , so that the parallax has no effect upon the aberration, and the value of κ obtained is $-0''.003$, corresponding to the value $20''.467$ of the aberration.

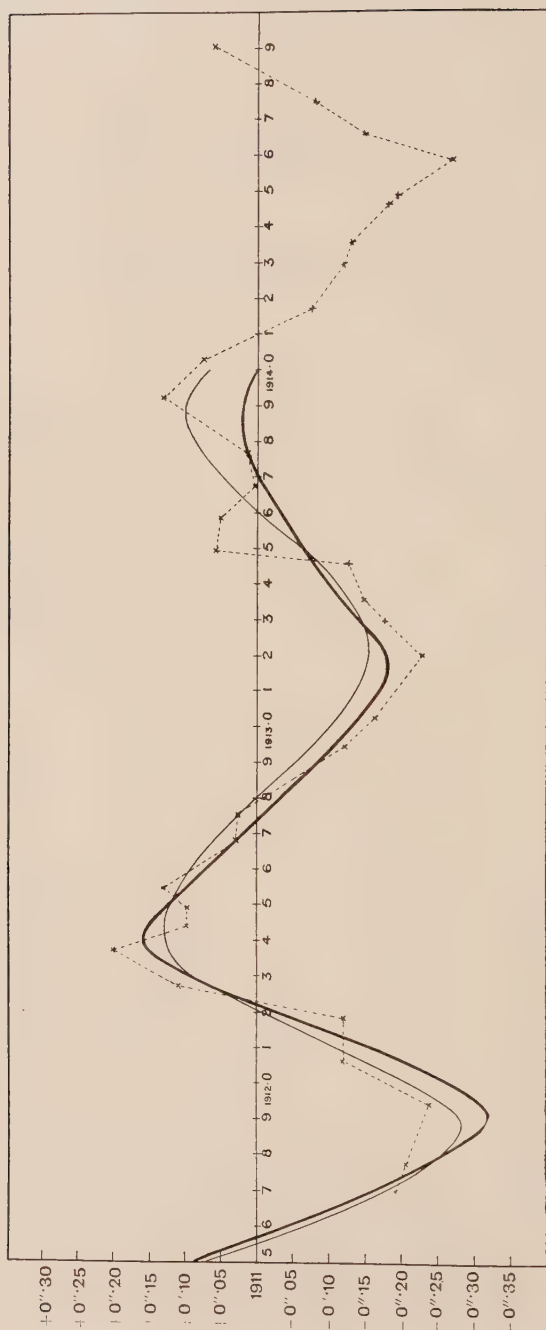
The normal equation for π gave the value $\pi = +0''.048$, but the weight of the equation is so small that no significance attaches to this result.

The values of the latitude variation and of the corrections to the assumed mean star places were substituted in the original equations of condition, and from the resulting residuals the probable error corresponding to unit weight was found to be $\epsilon = \pm 0''.056$. Calling e the probable error of the deduced value of the aberration, the probable errors of the z 's and t 's were obtained in terms of ϵ and e , and finally, from the normal equation for K , a relation was obtained between e and ϵ , viz. $9.76e = \epsilon$, so that the probable error of the aberration is almost exactly one-tenth of the probable error of an equation of condition corresponding to unit weight. Using the above value obtained for ϵ , we have $e = \pm 0''.0058$. The value of the aberration constant derived from rather more than three years' observations with the Cookson floating zenith-telescope is therefore

$$20''.467 \pm 0''.006.$$

From a knowledge of the proper motion, magnitude, and spectral type of each star, hypothetical individual parallaxes were written down, and the means for each group taken.* In Table II. is given in the first column the mean epochs to which the values of the latitude variation refer; in the second, the deduced

* For the following four stars, values of the parallax as actually determined were used:—1 Urs. Maj., Gp. v. $0''.03$, Yale; Gr. 1812, Gp. vi. $0''.03$, Kapteyn; χ Draconis, Gp. xii. $0''.116$, Greenwich (unpublished); 6 B Cygni, Gp. xiii. $0''.03$, Russell.



Comparison of the latitude variation, as given by the Cookson Telescope, with the results of the International Breitendienst.

(The international results are indicated by the thick continuous line.

The international results without the z term by the thin continuous line.

The Cookson results are plotted separately, and connected by a discontinuous line.)

values of the latter; in the third, these values corrected for parallax, using the hypothetical parallaxes; and in the last column, the probable errors of the values.

TABLE II.

Date.	Latitude Variation.		P.E.	Date.	Latitude Variation.		P.E.
	Uncorrected.	Corrected.			Uncorrected.	Corrected.	
1911 '70	" '121	" '114	± '016	1913 '45	" '067	" '049	± '019
'77	" '130	" '128	'015	'49	+ '121	+ '135	'025
'94	" '156	" '156	'017	'58	+ '121	+ '129	'020
1912 '06	" '041	" '041	'018	'67	+ '075	+ '082	'011
'18	" '042	" '038	'023	'76	+ '091	+ '092	'014
'27	+ '180	+ '190	'014	'92	+ '212	+ '212	'019
'37	+ '264	+ '279	'016	1914 '03	+ '155	+ '155	'017
'44	+ '161	+ '177	'016	'17	- '001	+ '005	'013
'48	+ '161	+ '176	'019	'29	- '052	- '041	'010
'54	+ '207	+ '216	'014	'35	- '064	- '049	'011
'68	+ '100	+ '107	'017	'46	- '116	- '098	'012
'76	+ '103	+ '105	'014	'49	- '133	- '118	'010
'94	- '039	- '038	'017	'58	- '199	- '188	'021
1913 '03	- '085	- '085	'019	'65	- '081	- '074	'014
'20	- '150	- '145	'025	'74	- '005	- '003	'014
'30	- '107	- '100	'018	'90	+ '140	+ '140	'013
'35	- '085	- '071	'016				

These results (in the sense $\phi_0 - \phi$) have been compared (up to 1914'0) with the results of the International Breitendienst. A constant correction of + 0"080 has been applied to the Cookson figures, for the purposes of direct comparison. The necessity for this correction arises from the fact that, since the star places used are based solely upon the observations during this epoch, they will give the mean latitude during this period, instead of the true latitude. On Plate 17 the international results are represented as a smooth curve; the Cookson results are plotted and connected by a discontinuous curve. The agreement is very satisfactory, the more so as the Cookson programme was designed primarily for the determination of the aberration, and the derivation of the latitude variation is more or less of a bye-product obtained in solving for the aberration. The 14-month period in the latitude variation is very accurately reproduced in the Cookson results.

The meaning of the z or Kimura term in the motion of the pole, as given in the international results, has been very fully discussed, without, at the same time, any indisputable origin having been assigned to it, and it is probable that the Cookson results will give some information on this point. The term has been

variously supposed to arise from small neglected terms in the nutation, from the parallax of the stars observed, or from the use of an incorrect aberration constant. Ross* has examined these suppositions, and found them capable of explaining only a small part of it, and he attributed it to a systematic refraction effect, due to disturbing conditions brought on at sunset, progressive throughout the night, and varying in amount with the season. In a later note † he states that "the Kimura term is a computational result, due to our ignorance of the correct method of partition of the closing sum" (Schlussfehler). The origin of the term and the question as to whether it possesses any physical significance are thus doubtful, but it is certain that any periodic errors in the system of declinations adopted will produce a term of the nature of the Kimura term. The Cookson latitude variation has, for these reasons, been compared also with the international results, after the z term has been subtracted from these (the resulting latitude variation being shown in the diagram by a thin continuous line). The agreement, on the whole, is improved, and more particularly during the latter half of 1914, when the z term attained its greatest value during this period. The international results for 1914-1915 are not yet available for comparison, and until then, further judgment must be suspended.

An examination of the co-latitude as furnished by the Cookson instrument is not without interest. The Cookson programme contains 21 pairs of stars, both of which are contained in Boss's *Preliminary General Catalogue*. The mean Z.D. north for each pair, subtracted from the mean declination of the pair for 1912.0, gives a value for the latitude. The Boss numbers of the stars used, their names, and the resulting latitudes are given in the following table, together with the number of plates from which the mean Z.D. is derived:—

Boss No.	Name.	Latitude 51° 28'.	No. of Plates.	Boss No.	Name.	Latitude 51° 28'.	No. of Plates.
668	π Persei	"		3385	Pi. XII. 255	"	
699	Lal. 5579	38°38	24	3415	Br. 1751	38°82	23
853	Gr. 731			3512	Br. 1791		
887	Lal. 7019	39°19	21	3530	Br. 1799	38°65	19
2101	Gr. 1385			3619	Pi. XIII. 296		
2142	Br. 1155	39°13	48	3667	ι Boötis	38°84	20
2404	ι Urs. Maj.			3827	Pi. XIV. 260		
2423	Gr. 1501	38°92	27	3854	Pulk. 2195	38°63	33
3093	Gr. 1825			3930	Rad. 3381		
3120	Br. 1609	38°50	22	3967	μ Cor. Bor.	38°57	29

* *Astr. Nach.*, 192, col. 133.

† *Astr. Nach.*, 193, col. 403.

Boss No.	Name.	Latitude No. of 51° 28'. Plates.		Boss No.	Name.	Latitude No. of 51° 28'. Plates.	
3985	Pi. XV. 153	"		4869	Br. 2416	"	
4022	Pulk. 2278	38° 87'	31	4893	Dpt. 2258	38° 62'	30
4138	σ Cor. Bor.	38° 34'	45	4980	Br. 2476	38° 35'	38
4213	Br. 2118			5005	Pi XIX. 211		
4270	Gr. 2377	38° 37'	46	5097	Br. 2552	38° 85'	33
4284	Br. 2149			5126	Gr. 3001		
4382	Pi. XVII. 63	38° 47'	38	5431	ξ Cygni	38° 86'	22
4411	Gr. 2435			5453	Pi. XXI. 51		
4672	χ Draco.	38° 75'	34	6071	λ Androm.	38° 71'	33
4702	Pulk. 2612			6105	Lal. 46607		
4727	Pi XVIII. 173	38° 82'	32				
4752	ζ ¹ Lyrae						

The mean value adopted for the latitude from these results is $51^{\circ} 28' 38''.69$, corresponding to a co-latitude of $38^{\circ} 31' 21''.31$. The Cookson telescope is situated 75 feet ($= 0''.75$) to the north of the transit circle, so that the corresponding co-latitude for the latter is $38^{\circ} 31' 22''.06$. To this a correction of $-0''.08$ must be applied, obtained from comparison of the Cookson results with those of the International Breitendienst, in order to reduce to the mean position of the Pole as adopted by them. The final co-latitude as deduced from the Cookson observations, upon the basis of Boss's system of declinations, is thus $38^{\circ} 31' 21''.98$. The value adopted from the transit-circle observations is $38^{\circ} 31' 21''.80$. With regard to the above values of the latitude provided by the separate pairs, it may be remarked that the discordances *inter se* are considerably larger than were considered *a priori* probable. They appear too large to be accounted for by errors in the adopted mean zenith distances of the various pairs, and at the same time they are larger than would be expected from the probable errors in the star places in Boss's Catalogue as given by Boss. The difference between the deduced co-latitude and the adopted transit-circle value may be partly due to a systematic error in the declinations of Boss's stars—such an error entering with full weight into the co-latitude. A comparison by Boss (*Astron. Journ.*, Nos. 531–2) of his catalogue with Newcomb's Fundamental Catalogue shows a mean systematic difference in declination for the zone of declination which includes the Cookson stars in the sense Boss–Newcomb $= -0''.18$. Also a comparison of Newcomb's Catalogue with Auwers' New Fundamental Catalogue of the *Berliner Jahrbuch* (1907) gives for the same zone,

Auwers - Newcomb = $-0''.12$. Hence the co-latitudes obtained, based on the three Fundamental Catalogues, are

Newcomb	.	.	.	.	.	$38^{\circ} 31' 21''.80$
Boss	.	.	.	.	.	$21'.98$
Auwers	.	.	.	.	.	$21'.92$

The stars in the Cookson programme are being put upon the Transit-Circle Working Catalogue, so that further material will be available for the discussion of the co-latitude when the series of observations with the Cookson instrument has been brought to a conclusion.

Royal Observatory, Greenwich:
1915 May 1.

The Greenwich R - D System. By W. G. Thackeray.

(Communicated by the Astronomer Royal.)

The system of reducing the zenith distances with the transit circle to the mean of reflection and direct observations has been much discussed and for various reasons not much followed at other observatories.

For some reason the R - D during the period 1897-1905 was much smaller than in previous years, but has increased again during the last nine years 1906-1914.

I propose to take these two periods of change as a test of the validity of the system. The reflection observations are classed as R, the direct observations taken in connection with the reflection observations as D, and the ordinary direct observations as D_0 .

We will consider first the general characteristics of the R - D discordance, and finally the best system of correction to be adopted.

1. The general characteristics we will consider in two sections:—

- (a) The general form and formula of the discordance for the two periods for the mean observer, and the separate values of the $b \sin z$ term for the regular observers.
- (b) The separate values of the formula $a + b \sin z$ for the R and D observations derived from a comparison with Boss's Preliminary Catalogue for 1900.

(a) In the first table is given the observed R - D for the two periods for every 4° of zenith distance for north and south stars. The values of a and b from the usual formula $a + b \sin z$ are

derived from the two quantities for north and south stars given in the same horizontal line, which depend on from fifty to two hundred observations each. The reason for thus solving them is in order to trace the validity of the formula $a + b \sin z$. The mean result is practically the same as the R-D solved in the usual way. The result for both periods is identical in form and shows that the value of b runs off rapidly as the zenith is approached.

The second table is derived from the figures printed in the tables of *Greenwich Observations*, 1911, p. A. xxvi. Arranging the figures there given for each observer in three groups, a fairly good value is obtained for each group for a solution similar to that for the mean observer. As the values of " a " do not give any discordances of note we will confine ourselves to the value of " b " alone. These show large differences for the different observers, but all the observers show the same characteristic that the mean observer showed in the general solution, the rapid diminution of " b " as the zenith is approached. This seems to indicate that the cause is due either to systematic differences in the R or D observations or in both.

TABLE I.

R-D Discordance and Values of $a + b \sin z$.

ZD	1897-1905.					1906-1914.				
	R-D.		Values of			R-D.		Values of		
	South.	North Stars.	South Stars.	a .	b .	North Stars.	South Stars.	a .	b .	b .
66°	- '30	+ '75	+ '22	+ '58		- 1 '17	+ 1 '23	+ '03	+ 1 '32	
62	- '60	+ '88	+ '14	+ '84		- 1 '12	+ 1 '15	+ '01	+ 1 '27	
					+ '70					+ '20
58	- '36	+ '79	+ '21	+ '69		- 0 '79	+ 1 '11	+ '16	+ 1 '13	
54	- '43	+ '68	+ '12	+ '69		- '77	+ 1 '01	+ '12	+ 1 '10	
50	- '32	+ '52	+ '10	+ '56		- '54	+ '77	+ '11	+ '85	
46	- '04	+ '52	+ '24	+ '39		- '43	+ '82	+ '19	+ '87	
					+ '43					+ '81
42	- '23	+ '38	+ '07	+ '46		- '41	+ '63	+ '11	+ '78	
38	- '03	+ '33	+ '15	+ '30		- '26	+ '63	+ '18	+ '73	
34	+ '02	+ '25	+ '13	+ '21		- '14	+ '43	+ '15	+ '51	
30	- '12	+ '28	+ '08	+ '40		- '20	+ '19	'00	+ '39	
					+ '27					+ '46
26	+ '12	+ '27	+ '19	+ '18		- '24	+ '28	+ '02	+ '60	
22	+ '09	+ '32	+ '20	+ '31		+ '05	+ '32	+ '18	+ '36	
Mean ...	...	...	+ '15	+ '47		...	...	+ '10	+ '98	

TABLE II.

Values of $b \sin z$ for Different Observers.

Limits of ZD.	H.	AC.	B.	B.	W.	JS.	W.	RC.	E.
	1897-1901.			1902-1905.			1906-1911.		

Values of b .

68°-52°	+1 ⁰⁰	+ ⁰¹	+ ⁴⁴	+ ⁷⁴	+ ⁸⁸	+ ³⁸	+1 ¹⁸	+1 ⁰⁴	+1 ⁷³
52-36	+ ⁷⁹	- ¹³	+ ³⁷	+ ⁵¹	+ ⁶⁸	+ ²³	+1 ⁰⁴	+ ⁶⁴	+1 ⁸³
36-20	+ ⁴²	- ⁶²	+ ³⁵	+ ³⁰	+ ⁴⁰	+ ¹⁴	+ ³⁸	+ ³⁸	+1 ¹⁵

(b) The following table is derived from a comparison of the R and D observations with Boss's Preliminary Catalogue for 1900. For the first period the Greenwich results are corrected to Pulkowa refractions and colatitude $21''\cdot80$ to make them comparable with the second period. In order to lighten the work of comparing with Boss, the results for all observers for the years 1899, 1900, and 1901 being found to give sufficient material for the purpose have been taken as representing the first period 1897-1905. The second period is represented by the observations of the regular observers only, and the observations are treated separately for each observer; this gives a further test of the personality of the observers.

The comparisons Boss - R and Boss - D are divided into two groups on each side of the zenith of roughly 20° zenith distance, and thus enable us to form in the same way as before new values for $a + b \sin z$ for R and D observations separately. It will be seen that while, as before, different observers differ considerably, the main characteristics of the R and D observations are quite different. While the R observations give a small value of b , increasing as the nadir is approached, the D observations give a large constant value of b . The peculiarity of the R - D discordance shown in section (a) is thus traced to the R observations.

It is also to be noticed that the comparison indicates that as the D observations support a pure flexure formula, and that the value of the flexure for the R observations is small as compared with the D observations, the best compromise will be to keep the R - D formula as $a + b \sin z$ and possibly adopt some other proportion than $\frac{R - D}{2}$ for the correction to the R and D observations.

TABLE III.

Comparison with Boss for R and D Observations separately for the Two Periods 1899-01 and 1906-11, and Corresponding Values of Formula $a + b \sin z$.

Period.	Limits of ZD.	North Stars.	South Stars.	<i>a.</i>	<i>b.</i>	North Stars.	South Stars.	<i>a.</i>	<i>b.</i>
R Observations.						D Observations.			
1899-01	60°-40°	+ "14	+ "23	+ "18	+ "06	- "08	+ "64	+ "27	+ "48
	40-20	- '03	+ '29	+ '13	+ '32	+ '07	+ '59	+ '33	+ '52
1906-II	60-40	- '05	'00	- '02	+ '03	- '43	+ 1 '13	+ '35	+ 1 '03
	40-20	+ '02	+ '38	+ '20	+ '36	- '14	+ '88	+ '37	+ 1 '02

TABLE III.—*continued.*

Comparison with Boss for R and D Observations separately for Different Observers for Period 1906-11, and Corresponding Values of Formula $a + b \sin z$.

R Observations.					D Observations.				
W.									
1906-11	60-40°	+ "05 ⁵⁶	- "10 ⁷³	- "02	- "10	- "49	+ 1'07	+ "34	+ 1'04
	40-20	- '16 ⁴⁵	+ '44 ⁴⁵	+ '14	+ '60	- '08 ⁴⁴	+ '98 ⁴⁴	+ '45	+ 1'06
RC.									
	60-40	- '32 ⁴⁶	+ '11 ⁵³	- '10	+ '27	- '30	+ 1'13	+ '40	+ '95
	40-20	+ '10 ³¹	+ '53 ³⁴	+ '31	+ '43	+ '12 ³¹	+ '98 ³⁴	+ '55	+ '86
E.									
	60-40	+ '18 ²⁷	- '12 ³¹	+ '03	- '20	- '51 ²⁷	+ 1'34 ³¹	+ '41	+ 1'20
	40-20	+ '16 ²³	+ '29 ²⁹	+ '22	+ '13	+ '45 ²³	+ '92 ²⁹	+ '23	+ 1'37
BE.									
	60-40	- '10 ²⁸	+ '14 ²⁸	+ '02	+ '15	- '42 ²⁸	+ '90 ²⁸	+ '24	+ '88
	40-20	- '03 ¹⁵	+ '26 ²⁷	+ '11	+ '29	- '15 ¹⁵	+ '62 ²⁷	+ '23	+ '77

The subscript figures denote the number of observations on which the values depend.

2. Now let us consider the question of the R-D correction to be applied. The problem before us is to reconcile as best we may three classes of observation, viz. R, D, D_0 , two sides of the zenith, and two periods. To make the comparison as simple as possible the quantities Boss-Greenwich on the south of the zenith are reduced to the mean N.P.D. (roughly 80° N.P.D.) corresponding to the position of the pole, on the north of the zenith to corrections to the adopted colatitude 21'80. In other words, on the south of the zenith we have to make the differences Boss-Greenwich for the periods as uniform as possible, and on the north of the zenith to make the difference for an adopted colatitude for the two periods as small as possible.

The R and D observations are the same as have been used in the previous tables, but the D_0 results are derived for the first period from the general catalogue difference to Boss (corrected to Pulkowa refractions and colatitude 21'80), and for the second period for south stars from clock stars only, and for north stars from the results of the colatitude investigations for the years 1906-1911.

The following table exhibits the differences Boss-Greenwich for the two periods on four different suppositions:—

- (1) Without R-D.
- (2) With a uniform value for "b" of + "60 sin z.
- (3) With the observed R-D divided equally between the R and D observations.
- (4) With the observed R-D divided between the R and D observations in the ratio of 1 to 2.

The value (4) for the R-D correction as $\frac{R-D}{3}$ to the R observations and of $\frac{2}{3}(R-D)$ to the D observations is here adopted as the result of this discussion because it gives the best agreement on both sides of the zenith for the two periods for the figures given by the comparison with Boss.

Values of R-D Corrections.

Period.	(2)	(3)	(4)
	R Observations.		
1899-01	- "08 - "44 sin z	- "08 - "14 sin z	- "05 - "04 sin z
1906-11	- "08 - "44 sin z	- "05 - "34 sin z	- "03 - "17 sin z
	D Observations.		
1899-01	+ "08 + "60 sin z	+ "08 + "30 sin z	+ "10 + "40 sin z
1906-11	+ "08 + "60 sin z	+ "05 + "50 sin z	+ "07 + "67 sin z

The formulæ for the R observations include the correction + "16 sin z for the inclination of the verticals.

TABLE IV.

Boss-Greenwich for Stars South and North of the Zenith (the latter representing Corrections to the Adopted Colatitude 21"80).

Period.	South Stars.			North Stars.		
	R.	D.	D ₀ .	R.	D.	D ₀ .
(1) Without R-D.						
1899-01	+ "19	+ "62	+ "57	+ "06	"00	- "24
1906-11	+ "22	+ 1"00	+ "76	- "01	- "28	- "27
(2) Uniform R-D.						
1899-01	+ "51	+ "14	+ "09	- "13	+ "29	+ "05
1906-11	+ "51	+ "55	+ "31	- "23	+ "04	+ "05
(3) Observed $\frac{R-D}{2}$.						
1899-01	+ "36	+ "36	+ "29	+ "05	+ "12	- "12
1906-11	+ "48	+ "62	+ "38	- "17	- "02	- "01
(4) Observed $\frac{R-D}{3}$ and $\frac{2}{3}R-D$.						
1899-01	+ "27	+ "25	+ "20	+ "08	+ "15	- "09
1906-11	+ "36	+ "49	+ "25	- "08	+ "05	+ "06

(A positive correction for south stars and a negative correction for north stars implies that the observed zenith distances are too small.)

All of these solutions contain discordances too large to be considered other than systematic. If we take each of the series of observations R, D, and D_0 on their merits as given by the above figures, we see that the R observations without correction of any kind give results perfectly accordant for the two periods: on the south side of the zenith the correction to Boss agrees with the equator point of Newcomb's Fundamental Catalogue, on the north side no correction is required to the adopted colatitude; we also see that the D_0 observations are nearly as consistent for the two periods, and that on both sides of the zenith they can be made by a suitable flexure correction to harmonise with the "R" observations, or that the "R" and " D_0 " observations can both be made to harmonise in a common system; but when we come to the D observations we see that they are liable to much more considerable inconsistencies. This may probably be due to the nature of the observation, which is made hurriedly directly after the R observation. It therefore follows that as the R and D_0 observations each give consistent results, any basis of correction depending on reflection observations would be better derived from observations made independently on separate nights rather than on observations made on the same night.

The Large Eruptive Prominence of 1915 April 19.

By C. P. Butler, A.R.C.Sc. (Plate 18.)

(Communicated by the Director of the Solar Physics Observatory, Cambridge.)

With the rapid increase of the solar activity, prominences have been numerous during the last few months, many of them large and eruptive. The most interesting of these was photographed in $K_{2.3.2}$ light with the spectroheliograph of the Solar Physics Observatory, Cambridge, on 1915 April 19. The sky was partly cloudy, and the first plate was not obtained until 10^h 40^m a.m. On developing this, it was seen that an eruption of unusual extent was in progress, and efforts were made to obtain further records. Owing to the Sun being obscured, it was not possible to secure another plate until 11^h 30^m a.m., and from this it was seen that the arch had entirely disappeared, the only remnants being the two basal portions shown on the second photograph.

The two chief areas of activity were on the north-west and south-east limbs at points approximately diametrically opposite, the larger prominence of the present note being that on the south-east. This is in the form of an extensive arch, one end lying at about position-angle 115°-123°, the other at about position-angle 143°-153°. The former is much more intense and shows signs of regular sinuosity in the component filaments. The more southerly end shows much delicate detail, and gives the impression of falling material.

Extending from the northern end there is a mass of delicate tracery suggesting matter in rotation during its ascent, gradually curving over to the south and connected to the highest portions of the arch by fainter filaments. The highest portions consist of two large masses, one roughly parallel to the Sun's limb, the other dipping towards the limb, the drooping end being connected by more fine filaments to the southern end.

The extreme height of the arch measured from the limb is one quarter of the solar diameter, about $480''$, equivalent to 216,100 miles.

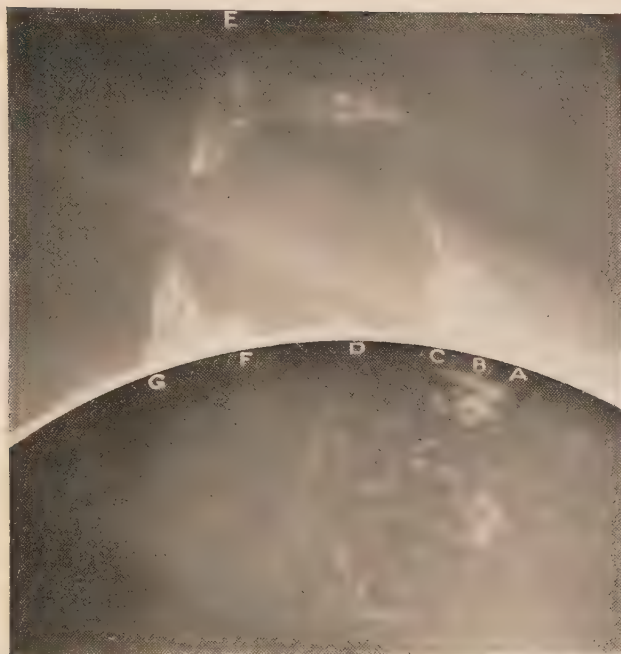
The low small prominence in the form of a shallow arch between the two chief ends has undergone little change during the interval between the two pictures, and probably was not connected with the chief eruption.

Details of Eruptive Prominence 1915 April 19.

		P. A. from N. pole.	Height.	
			10 ^h 43 ^m a.m.	11 ^h 41 ^m a.m.
Northern End	{ A	115°	54	72
	{ B	119	108	...
	{ C	123	126	...
	D	131	54	36
Highest Point	{ E	139	480	...
Southern End	{ F	143	54	54
	{ G	153	180	...

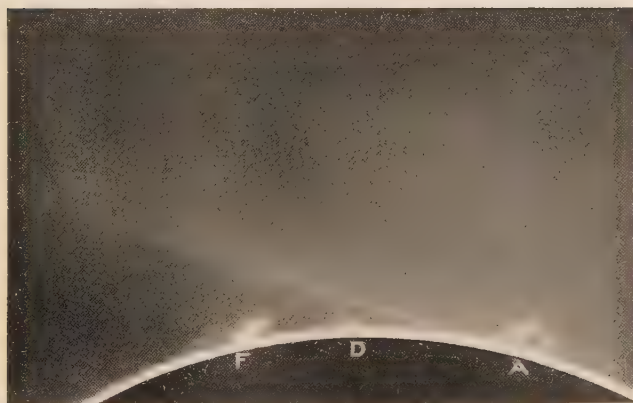
Association with the activity shown by Flocculi.—A comparison of the disc photograph in K_{2.3.2} light taken some minutes previously with others taken on April 20 and 21, furnishes interesting suggestions. The strongest group of flocculi visible on the Sun at the time is quite close to the limb at the same position-angle as the brighter end of the large prominence.

Immediately preceding this group of flocculi near the limb there is a widely distributed mass of smaller flocculi, and among these was rapidly developed during the two following days a group of large spots. This suggested to Professor Newall that the falling in of the prominence matter on the 19th might be the precursor of the surface disturbance which culminated in the formation of this spot group; the position of the spot area was plotted on the disc of the 19th, and on the assumption that the prominence was in the plane of the great circle that passes through the bright flocculi of the 19th and the neighbouring regions of the spot outburst of the 20th, a geometrical determination was made of the projection of the prominence arch to ascertain if it could be regarded as terminating in the spot area, by continuing its path in front of the disc within the limb. It was concluded that the



10^h 43^m

10^h 32^m 35^s



11^h 41^m

Large Eruptive Prominence, 1915 April 19, Solar Physics Observatory,
Cambridge.

probable track of the prominence matter could not be satisfactorily associated with the area where the spots were seen. This was on the assumption that there had been no drift of the spot area, the positions being referred back in longitude by simply accounting for the rotation of the zone for the particular latitude of the spot outburst.

The thanks of the writer are due to Professor Newall for advice and help in improving the conditions of observation, and to Mr. Manning for help in the preparation of the photographs for reproduction.

New Double Stars. By the Rev. T. E. Espin, M.A.

No.	B.D.	R.A.	1900.	Decl.	P.	D.	Mags.	Nts.	Date.
		h	m						1915.
1357	+44°	1	0 2'6	+44 34	138°1	5"14	8'7 13'5	1	'022
1358	+44°	129	32'1	+44 26	9'9	5'99	9'0 11'7	2	'048
1359	+44°	154	39'1	+44 45	330'6	2'49	11'0 11'2	2	'031 BC
					269'7	38'42	A=9'5	2	'031 AB
1360	+44°	242	1 2'4	+44 48	261'1	2'77	9'5 9'8	2	'048
1361	+44°	362	39'8	+45 4	268'5	2'90	12'2 12'4	3	'039 BC
					7'2	18'43	A=9'3	3	'039 AB
1362	+44°	401	54'2	+44 56	291'9	8'92	9'5 10'8	2	'031
1363	+44°	605	2 52'6	+44 38	205'1	5'55	9'5 10'4	3	'036
1364	+44°	653	3 11'3	+44 10	78'5	2'33	9'2 12'7	2	'086
1365	+44°	785	37'7	+44 16	230'4	4'69	9'3 10'7	3	'098
1366	+43°	878	56'2	+43 59	341'7	2'48	9'2 12'5	3	'119
1367	+43°	890	59'1	+43 56	12'0	6'68	8'2 13'2	2	'132
1368	+43°	915	4 3'2	+43 59	361'3	2'75	9'8 13'5	2	'140
1369	+43°	920	5'2	+43 57	207'2	6'98	9'1 12'5	2	'139
1370	+43°	974	20'3	+43 50	244'8	3'16	9'1 11'7	2	'171 AB
					153'5	7'01	C 14'0	1	'142 AC
1371	+43°	1039	34'3	+43 36	340'5	5'69	9'5 9'5	2	'171
1372	+44°	1063	50'8	+44 57	93'5	4'57	9'5 11'2	2	'031
1373	+44°	1130	5 5'9	+44 22	78'3	5'46	9'0 10'1	2	'094
1374	+44°	1142	7'7	+44 26	262'3	1'91	9'5 14'0	4	'088
1375	+44°	1174	13'3	+44 46	234'0	3'17	9'5 10'5	2	'031
1376	+44°	1196	18'7	+44 35	250'1	3'98	9'3 9'6	3	'182
1377	+44°	1204	20'6	+44 36	187'6	4'37	9'3 9'7	2	'168
1378	+44°	1284	40'6	+44 19	281'0	4'70	9'5 9'5	2	'164

No.	B.D.	R.A.	1900.		Decl.	P.	D.	Mags.	Nts.	Date.
			h	m						
1379	+44°	1305	5	46 ²	+44 33	297 ⁶	5 ⁴⁵	9 ⁷ 10 ⁸	3	1915. 237
1380	+44°	1341		55 ¹	+44 6	356 ¹	5 ²⁶	9 ⁵ 11 ⁹	3	'213
1381	+44°	1395	6	6 ⁰	+44 55	87 ³	1 ⁴⁰	9 5 9 ⁶	3	'043
1382	+44°	1416		12 ³	+44 39	358 ⁵	1 ⁸²	9 ⁵ 11 ⁷	3	081
1383				37 ⁰	+44 20	308 ⁴	1 ⁸⁸	9 ⁵ 9 ⁵	3	'169
1384	+43°	1591		37 ²	+43 41	310 ¹	3 ²⁹	9 ⁵ 10 ⁶	3	'213
1385				42 ⁴	+44 11	255 ⁵	1 ⁴⁷	9 ² 10 ⁵	5	'213
1386	+42°	1746	7	33 ³	+42 20	64 ⁹	9 ⁰³	8 ³ 9 ⁰	1	'257
1387	+43°	1760		53 ⁵	+42 59	140 ⁸	5 ¹¹	9 ⁵ 10 ²	3	'223
1388	+43°	1803	8	13 ⁶	+43 4	176 ⁸	3 ⁷²	9 ⁵ 14 ⁰	3	'235
1389	+43°	1829		28 ²	+43 21	170 ³	4 ⁰³	9 ⁷ 11 ⁷	2	'195
1390	+43°	1865		49 ⁹	+43 26	190 ⁷	1 ⁵¹	9 ⁴ 9 ⁷	2	'246
1391	+43°	1884	9	2 ³	+42 54	283 ⁷	5 ⁵⁹	9 ⁵ 9 ⁶	2	'253
1392	+43°	1906		14 ⁷	+43 26	271 ⁷	7 ⁴⁹	9 ³ 11 ⁷	2	'246
1393	+42°	2059		47 ⁵	+42 41	215 ⁴	3 ⁴¹	9 ⁵ 14 ⁰	3	'268
1394	+41°	2080	10	18 ⁰	+41 4	141 ⁷	6 ²⁰	8 ⁵ 9 ⁶	3	'339
1395	+43°	2022		23 ⁴	+43 4	76 ⁰	4 ³⁵	9 ⁰ 10 ⁰	2	'246
1396	+42°	2129		28 ⁰	+42 29	186 ⁹	5 ¹⁴	9 ⁴ 12 ⁵	2	'316
1397				39 ³	+40 54	140 ¹	4 ⁴³	9 ⁸ 12 ⁵	2	'332
1398				57 ⁴	+41 5	335 ⁵	2 ⁷²	9 ⁴ 13 ³	3	'340
1399	+42°	2213	11	24 ⁹	+41 49	302 ³	3 ¹⁴	9 ⁷ 9 ⁸	2	'278
1400	+41°	2202		27 ⁵	+41 40	180 ⁵	2 ⁸²	9 ⁴ 13 ⁰	2	'322
1401	+41°	2205		29 ²	+41 25	325 ⁷	6 ⁶¹	9 ⁷ 10 ⁰	3	'336
1402	+41°	2315	12	34 ⁸	+41 1	23 ⁶	3 ⁵⁰	9 ³ 10 ³	3	'339
1403	+41°	2325		38 ⁹	+40 48	215 ¹	2 ⁷⁵	9 ³ 10 ²	3	'339
1404	+40°	2584		47 ³	+40 44	30 ⁶	2 ⁷⁷	9 ⁵ 13 ⁵	2	'345
1405				48 ⁰	+40 22	254 ⁷	3 ⁹²	10 ⁰ 10 ²	1	'359

Notes.

1366.—*Comes*, 13 mag., P 281°, D 25°.

1385.—Professor Jonckheere kindly looked up this object with the 28 in. at Greenwich and measured it:—

1915 March 26 P 229°·1, D 1°·41, Mags., 9·4, 11·3.

1405.—This pair forms a distant companion to B.D.+40° 2585 at P 284°·4, D 66°.

Dates of Maximum of S Herculis, with New Elements. By the
Rev. T. E. R. Phillips, M.A. (Plate 19.)

The long-period variable S Herculis is of interest, apart from the nature of its light curve, inasmuch as the dates of maximum are seemingly irregular, and also appear to be affected by some periodic inequality. In his third Catalogue Chandler gave as the period of the star the simple value of 308.1 days with the note "large irregularities," but in his revised Catalogue a sine term was added, the formula being

$$2399.197 + 308.3E + 35 \sin(9^\circ E + 86^\circ).$$

In vol. lv. of the *Memoirs* of the Society Professor Turner compared the Rousdon observations with Chandler's revised formulæ, and pointed out that, although the new formulæ were generally found to be a great improvement on the old, the case of S Herculis was a striking exception.* Later the observations of this star by the members of the Variable Star Section of the British Astronomical Association were published (*B.A.A. Memoirs*, vols. xv. and xviii.), and subsequently the individual light curves based on the work of the Section down to the end of 1909 (Appendix to vols. xv. and xviii.). This data has been further supplemented through the kindness of Mr. C. L. Brook, the present Director of the Section, in supplying privately the unpublished means of the Section's observations of S Herculis from 1910 to the end of 1914. Quite recently the editing and publishing of Baxendell's observations by Professor Turner and Miss Blagg (*M.N.*, vol. lxxv., No. 5, p. 398) has provided additional material of an early date. Baxendell's observations are in two groups separated by an interval of about nine years. This interval is to be regretted as, apart from this, we now have an almost complete record from 1857 to the present time. At the conclusion of their paper Professor Turner and Miss Blagg tabulate a number of solutions for the dates of maximum, which, however, were worked out without making use of the B.A.A. results. The present paper is an attempt to find a solution which will satisfy the whole of the material now available.

1. In a discussion of this sort it seems desirable to consider first what is to be taken as the date of maximum of a variable star, for there are two possible alternatives: (1) the date on which the star appears to have been actually brightest; (2) the date indicated by considerations of symmetry with regard to the star's rise and decline as determined by the gradient at other parts of the curve.

* The error of Chandler's formula is now ($E=70$), about +120 days. Guthnick's formula, given in Hartwig's Ephemeris, is: $-2399.219 + 307.5E + 47 \sin(7^\circ.5 E + 120^\circ) + 12 \sin(15^\circ.0 E + 60^\circ)$, and its present error about +50 days.

The former and perhaps the more natural plan seems open to the objections: (a) that the highest point on a light curve is often very difficult to determine (see remarks by Professor Turner, *M.N.*, vol. lxvii., No. 5, p. 337 *et seq.*), and (b) that observations near maximum are frequently liable to error owing to the star's redness or the want of suitable comparison stars in the field. The alternative plan (2) has the advantages (a) of representing the curve more or less as a whole, and (b) that, provided a sufficient portion of the curve is available, the probable date of maximum (compared with the adopted maximum of the mean curve) can be computed when the star was unobserved or unobservable about that time. The formula for correction to epoch given by Professor Turner in *M.N.*, vol. lxxiii., No. 2, p. 134, is

$$\frac{N}{2\pi} \Sigma D(p-f) / \Sigma (p-f)^2,$$

where N = the adopted period of the star, D the residual at any point referred to the mean light curve at that point, $p-f$ = the difference between the values of the ordinates preceding and following the ordinate in question, as read from the mean curve.

In Table I. are brought together and compared the dates of maximum obtained by both methods. Column 1 gives the epoch or number of the period; column 2, the "observed" dates of maximum, or the dates as read from the plotted curves in the neighbourhood of maximum; column 3, the dates of maximum computed by the formula above given. In column 2 the dates of epochs 1 to 34 are estimated from Baxendell's observations published by Professor Turner and Miss Blagg (*M.N.*, vol. lxxv., No. 5, p. 413 *et seq.*); those of epochs 37 to 51 are similarly estimated from the Rousdon observations in vol. lv., *R.A.S. Memoirs*, p. 90 *et seq.*, and the remaining dates are obtained from the light curves drawn from the B.A.A. Variable Star Section's observations. In column 3 the dates of epochs 1 to 34 are taken from the paper by Professor Turner and Miss Blagg (vol. lxxv., No. 5, p. 420); those of epochs 37 to 52 have been computed from the Rousdon residuals published in vol. lv. of the *Memoirs* of the Society, pp. lxxxv and lxxxvi, and those of epochs 58 to 69 are computed from the observations of the B.A.A. Variable Star Section. Column 4 gives the difference between the two sets of dates, and it will be seen that although there is a general agreement which one would expect, there are sometimes discrepancies amounting to nearly 20 days.

TABLE I.

Dates of Maximum of S Herculis.

E.	"Observed" Dates. J. D.	Computed Dates. J. D.	O - C.	E.	"Observed" Dates. J. D.	Computed Dates. J. D.	O - C.
1	2399565	2399568.9	- 3.9	43	...	2477.4	...
2	9879	(9876.9)	+ 2.1	44	2780	(2789.7)	- 9.7
3	2400183	2400177.8	+ 5.2	45	3098	(3099.4)	- 1.4
6	...	1077.9	...	46	3408	3405.4	+ 2.6
7	1379	1376.4	+ 2.6	47	3720	3715.0	+ 5.0
8	1684	1678.5	+ 5.5	48	...	(4023.2)	...
9	...	(1983.0)	...	49	...	4332.4	...
10	...	(2279.7)	...	50	4631	(4611.9)	+ 19.1
11	...	(2572.0)	...	51	4922	4922.2	- 0.2
12	2877	2874.9	+ 2.1	52	5238	5241.1	- 3.1
13	3185	(3179.8)	+ 5.2	53	5542	...	...
14	3476	...	...	56	6433	...	...
.	.	.	.	57	6738	...	...
26	(7212)	(7208.0)	+ 4.0	58	7049	7049.4	- 0.4
27	...	(7521.0)	...	59	7345	7344.0	+ 1.0
28	7826	...	...	60	7661	7649.7	+ 11.3
29	...	8137.6	...	61	7963	7954.7	+ 8.3
30	...	(8456.4)	...	62	8250	8252.9	- 2.9
33	(9383)	9394.9	- 11.9	63	8562	8560.6	+ 1.4
34	9701	9707.5	- 6.5	64	(8867)	8864.3	+ 2.7
37	2410615	(2410622.9)	- 7.9	65	9155	9157.1	- 2.1
38	0925	0926.5	- 1.5	66	(9433)	9451.5	- 18.5
39	1239	1227.6	+ 11.4	67	9761	9753.8	+ 7.2
40	1560	1541.7	+ 18.3	68	2420047	2420055.6	- 8.6
41	(1865)	(1847.3)	+ 17.7	69	0350	0357.1	- 7.1
42	...	2162.9	...				

N.B. —The figures in brackets are somewhat uncertain owing to the paucity of observations or other reasons.

2. An alternative method of discussion would be to make use of the dates of minimum, but the curve is usually rather flat near that point, and the faintness of the star at such times placed it beyond the range of Baxendell's instruments.

3. The question now arises as to the homogeneity of the above material. Are the different series of observations mutually comparable, or are they appreciably affected by some kind of personal equation? Unfortunately we have no means of directly comparing

Baxendell's with the Rousdon series, but the Rousdon observer,* Mr. C. Grover, has very kindly forwarded privately his observed dates of maximum since those published in vol. lv. of the *Memoirs*, and these can be compared with those taken from the light curves of the B.A.A. Variable Star Section. The result of the comparison is shown in Table II.

TABLE II.
"Observed" Dates of Maximum of *S Herculis*.

E.	Rousdon. J. D.	B.A.A. J. D.	Rousdon, B.A.A.	E.	Rousdon. J. D.	B.A.A. J. D.	Rousdon, B.A.A.
52	2415238	2415238	0	63	8550	8562	-12
53	5521	5542	-21	64	8865	(8867)	-2
57	6746	6738	+8	67	* 9739	9761	-22
58	7035	7049	-14	68	2420029	2420047	-18
59	7345	7345	0	69	0325	0350	-25
62	8236	8250	-14				

It will be seen that the differences, especially towards the end of the series, are sometimes very considerable. The idea suggests itself that possibly there is a change of colour in the star at or after its rise to maximum. To an inquiry on this point Mr. Grover replied that, as is the case with many other long-period variables, the red colour is palest at maximum and deepens as the light wanes. When about 9 m. or less it is often of a very deep ruddy tone. There are, however, variations from the general rule, *e.g.* Mr. Grover says that in 1887, 1888, and 1889 the star was fiery red at maximum, and in 1902 deep orange. In other years it was pale yellow or almost white. Reference may be made here to the case of *T Cephei*, discussed by Professor Turner in vol. lv., *Memoirs*, p. civ, and *Memoirs, B.A.A.*, vol. xviii. p. 6 *et seq.*, from which it is abundantly clear that under certain circumstances observers may differ among themselves at the same magnitudes according as the star is on the rise or fall. This may be due to some difference of colour at the two parts of the curve; and *S Herculis* may present an analogous case. The question of the telescope used—whether a reflector or refractor, and if the latter, the nature of its colour correction—should also be considered, but there is not the material for a discussion of the point here.

4. The question must now be settled as to how the dates in Table I. are to be treated in the final analysis. Are columns 2 and 3 to be somehow combined, or is one of them to be chosen in preference to the other, and if so which? On consideration a method of compromise seemed unsatisfactory, and it is perhaps

* After this was written I received from Mr. Grover a diagram of his observations of the star in 1886, together with those of Baxendell. These were Mr. Grover's first and Baxendell's last observations of this star. The accordance is satisfactory, but the observations of Baxendell are too few and scattered to establish any comparison between the habit of the two observers.

desirable to find a solution for each series separately. But there still remains the question of the discrepancy between Rousdon and the B.A.A. Variable Star Section. The mean value of Rousdon—B.A.A. is -11 , but this is largely due to the last three residuals at $E=67, 68$, and 69 . Neglecting these, the addition of 6 days to the observed Rousdon dates will practically bring them into line with the B.A.A. series, and on the whole it seems preferable to take the latter as the standard since they are based upon the work not of one but of a number of observers.

As regards the computed dates in column 3 of Table I. it is not at first so clear that the same correction of $+6$ days should be applied, for it might be supposed that the discrepancy is a peculiarity connected with the star's appearance at maximum. To test this point the mean light curves of the Baxendell, Rousdon, and B.A.A. series are brought together as follows:—

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
Baxendell	11.7	(11.9)	11.3	10.2	9.3	8.6	8.0	7.7	8.3	9.2	10.2	11.0
Rousdon	12.0	11.8	11.0	10.3	9.7	8.6	7.9	7.6	8.1	9.1	10.1	11.4
B.A.A.	12.0	12.0	11.0	9.8	8.8	8.1	7.6	7.7	8.8	9.1	10.9	11.7

Now on plotting these it is found that there is no evidence whatever of an accelerated Rousdon maximum as compared with the other two curves. It is accordingly concluded (taking the B.A.A. curve as standard) that in the observations the Rousdon curve is shifted bodily, and that the correction $+6$ days must be applied to the computed as well as to the "observed" dates to make the material homogeneous.

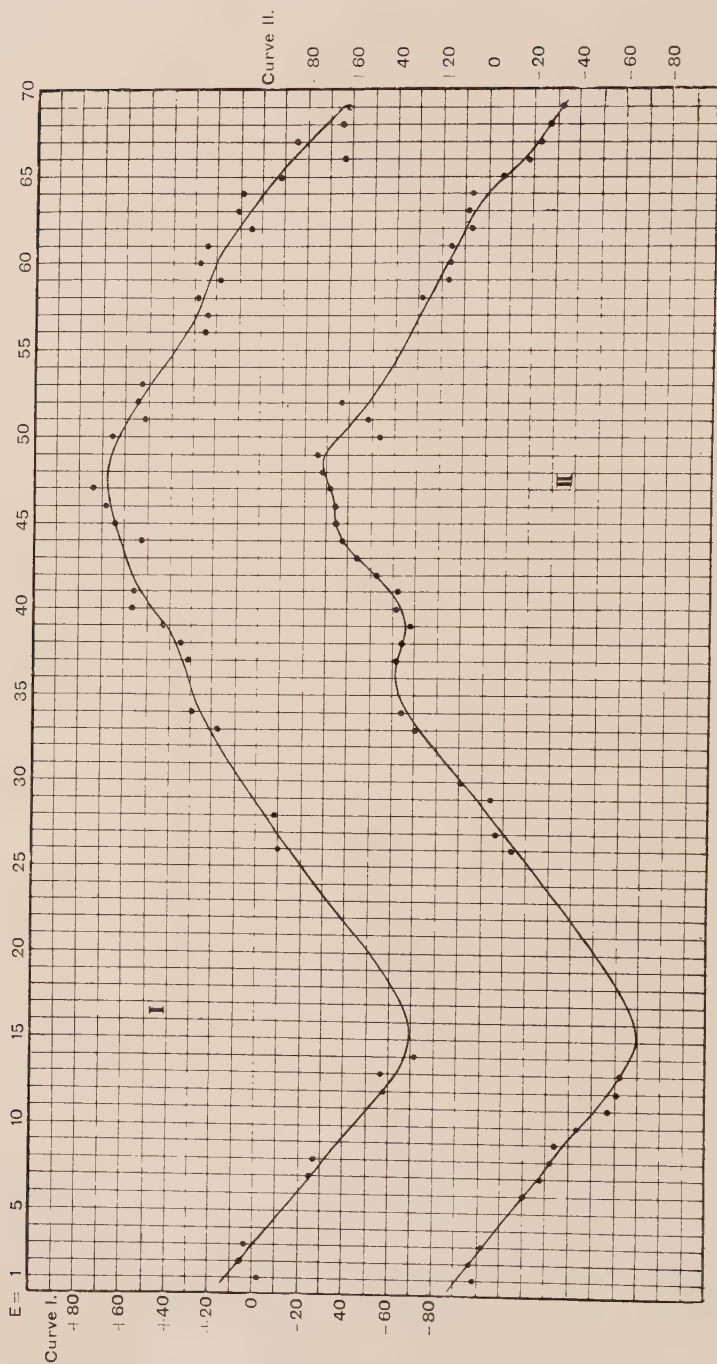
6. Turning now to the analysis of the dates, the first question is that of the mean period. The periods for Baxendell's two series ($E=1$ to 13 , and $E=26$ to 34), as given by Professor Turner and Miss Blagg (*M.N.*, vol. lxxv. pp. 420 and 422), are 300.1 and 312.5 days respectively. Professor Turner found for the Rousdon series at $E=41$, $P=310.3$ days; and at $E=46$, $P=307.6$ days, or 309 days in the mean. The B.A.A. observations give for $E=58$ to 61 , $P=302.7$ days; for $E=62$ to 65 , $P=300.5$ days; and for $E=66$ to 69 , $P=299.5$ days. The value 300.1 days given by Professor Turner and Miss Blagg applies to Baxendell's first series as a whole: but making use of the available residuals on p. 419 of *M.N.*, vol. lxxv., No. 5, we get for $E=1$ to 7 , $P=300.6$ days, and for $E=7$ to 14 , $P=298.4$ days. These figures are perhaps not very reliable owing to the scantiness of material available for such a determination, but on the whole it seems likely that the period of the star was about the same at $E=3$ and $E=64$. Taking, then, the available pairs of observations at intervals of 61 epochs, we get from both series of maxima a mean value of 306.2 days, which is probably not far from the truth.

7. In Table III. the material for analysis is assembled and compared with a uniform period of 306.2 days; but the correction

of +6 days, as found above, has been applied to the Rousdon figures in columns 2 and 3 of Table I. for E=37 to 51, and E=37 to 52 respectively.

TABLE III.

E.	"Observed" Date. J.D.	Computed Date. J.D.	Date from Uniform Period of 306 ¹ / ₂ days. (2399260 ¹ / ₈ +306 ¹ / ₂ E.)	O - Uniform Period.	C - Uniform Period.
1	2399565	2399568 ⁹ / ₉	2399567 ⁰ / ₀	- 2 ⁰ / ₀	+ 1 ⁹ / ₉
2	9879	(9876 ⁹ / ₉)	9873 ² / ₂	+ 5 ⁸ / ₈	+ 3 ⁷ / ₇
3	2400183	2400177 ⁸ / ₈	2400179 ² / ₂	+ 3 ⁸ / ₈	- 1 ⁴ / ₄
6	...	1077 ⁹ / ₉	1098 ⁰ / ₀	...	- 20 ¹ / ₁
7	1379	1376 ⁴ / ₄	1404 ² / ₂	- 25 ² / ₂	- 27 ⁸ / ₈
8	1684	1678 ⁵ / ₅	1710 ⁴ / ₄	- 26 ⁴ / ₄	- 31 ⁹ / ₉
9	...	(1983 ⁰ / ₀)	2016 ⁶ / ₆	...	- 33 ⁶ / ₆
10	...	(2279 ⁷ / ₇)	2322 ⁸ / ₈	...	- 43 ¹ / ₁
11	...	(2572 ⁰ / ₀)	2629 ⁰ / ₀	...	- 57 ¹ / ₁
12	2877	2874 ⁹ / ₉	2935 ² / ₂	- 58 ² / ₂	- 60 ³ / ₃
13	3185	(3179 ⁸ / ₈)	3241 ⁴ / ₄	- 56 ⁴ / ₄	- 61 ⁶ / ₆
14	3476	...	3547 ⁶ / ₆	- 71 ⁶ / ₆	...
26	(7212)	(7208 ⁰ / ₀)	7222 ⁰ / ₀	- 10 ⁰ / ₀	- 14 ⁰ / ₀
27	...	(7521 ⁰ / ₀)	7528 ² / ₂	...	- 7 ² / ₂
28	7826	...	7834 ⁴ / ₄	- 8 ⁴ / ₄	...
29	...	8137 ⁶ / ₆	8140 ⁶ / ₆	...	- 3 ⁰ / ₀
30	...	(8456 ⁴ / ₄)	8446 ⁸ / ₈	...	+ 9 ⁶ / ₆
33	(9383)	9394 ⁹ / ₉	9365 ⁴ / ₄	+ 17 ⁶ / ₆	+ 29 ⁵ / ₅
34	9701	9707 ⁵ / ₅	9671 ⁶ / ₆	+ 29 ⁴ / ₄	+ 35 ⁹ / ₉
37	2410621	(2410628 ⁹ / ₉)	2410590 ² / ₂	+ 30 ⁸ / ₈	+ 38 ⁷ / ₇
38	0931	(0932 ⁵ / ₅)	0896 ⁴ / ₄	+ 34 ⁶ / ₆	+ 36 ¹ / ₁
39	1245	1233 ⁶ / ₆	1202 ⁶ / ₆	+ 42 ⁴ / ₄	+ 31 ⁰ / ₀
40	1566	1547 ⁷ / ₇	1508 ⁸ / ₈	+ 57 ² / ₂	+ 38 ⁹ / ₉
41	(1871)	(1853 ³ / ₃)	1815 ⁰ / ₀	+ 56 ⁰ / ₀	+ 38 ³ / ₃
42	...	2168 ⁹ / ₉	2121 ² / ₂	...	+ 47 ⁷ / ₇
43	...	2483 ⁴ / ₄	2427 ⁴ / ₄	...	+ 56 ⁰ / ₀
44	2786	(2795 ⁷ / ₇)	2733 ⁶ / ₆	+ 52 ⁴ / ₄	+ 62 ¹ / ₁
45	3104	(3105 ⁴ / ₄)	3039 ⁸ / ₈	+ 64 ² / ₂	+ 65 ⁶ / ₆
46	3414	3411 ⁴ / ₄	3346 ⁰ / ₀	+ 68 ⁰ / ₀	+ 65 ⁴ / ₄
47	3726	3721 ⁰ / ₀	3652 ² / ₂	+ 73 ⁸ / ₈	+ 68 ⁸ / ₈
48	...	(4029 ² / ₂)	3958 ⁴ / ₄	...	+ 70 ⁸ / ₈
49	...	4338 ⁴ / ₄	4264 ⁶ / ₆	...	+ 73 ⁸ / ₈
50	4637	(4617 ⁹ / ₉)	4570 ⁸ / ₈	+ 66 ² / ₂	+ 47 ¹ / ₁
51	4928	4928 ² / ₂	4877 ⁰ / ₀	+ 51 ⁰ / ₀	+ 51 ² / ₂



Dates of Maximum of S Herculis compared with a uniform period of 306.2 days.

TABLE III.—*continued.*

E.	"Observed" Date. J.D.	Computed Date. J.D.	Date from Uniform Period of 306 ¹ / ₂ days. (2399260 ¹ / ₈ + 306 ¹ / ₂ E.)	O - Uniform Period.	C - Uniform Period.
52	2415238	2415247 ¹ / ₁	2415183 ¹ / ₂	+ 54 ¹ / ₈	+ 63 ¹ / ₉
53	5542	...	5489 ¹ / ₄	+ 52 ¹ / ₆	...
56	6433	...	6408 ¹ / ₀	+ 25 ¹ / ₀	...
57	6738	...	6714 ¹ / ₂	+ 23 ¹ / ₈	...
58	7049	7049 ¹ / ₄	7020 ¹ / ₄	+ 28 ¹ / ₆	+ 29 ¹ / ₀
59	7345	7344 ¹ / ₀	7326 ¹ / ₆	+ 18 ¹ / ₄	+ 17 ¹ / ₄
60	7661	7649 ¹ / ₇	7632 ¹ / ₈	+ 28 ¹ / ₂	+ 16 ¹ / ₉
61	7963	7954 ¹ / ₇	7939 ¹ / ₀	+ 24 ¹ / ₀	+ 15 ¹ / ₇
62	8250	8252 ¹ / ₉	8245 ¹ / ₂	+ 4 ¹ / ₈	+ 7 ¹ / ₇
63	8562	8560 ¹ / ₆	8551 ¹ / ₄	+ 10 ¹ / ₆	+ 9 ¹ / ₂
64	(8867)	8864 ¹ / ₃	8857 ¹ / ₆	+ 9 ¹ / ₄	+ 6 ¹ / ₇
65	9155	9157 ¹ / ₁	9163 ¹ / ₈	- 8 ¹ / ₈	- 6 ¹ / ₇
66	(9433)	9451 ¹ / ₅	9470 ¹ / ₀	- 37 ¹ / ₀	- 18 ¹ / ₅
67	9761	9753 ¹ / ₈	9776 ¹ / ₂	- 15 ¹ / ₂	- 22 ¹ / ₄
68	2420047	2420055 ¹ / ₆	2420082 ¹ / ₄	- 35 ¹ / ₄	- 26 ¹ / ₈
69	0350	0357 ¹ / ₁	0388 ¹ / ₆	- 38 ¹ / ₆	- 31 ¹ / ₅

8. Plotting the residuals in columns 5 and 6 of the above table we obtain the curves I. and II. (see Plate 19), in regard to which the following points seem worth noticing:—

- (a) The periodic inequality, which is often masked by minor irregularities when only a few periods are considered, is established beyond question. This is of interest in view of the doubt which has sometimes been cast on such alleged inequalities.
- (b) The inequality is obviously not a simple sine term, but is of considerable complexity.
- (c) Accidental errors, if we may so call them, are greatly reduced when the dates of maximum are calculated from the curve. Save for the apparent discrepancies in curve II. at E=50, 51, and 52, which may very possibly be due to a real kink at that part, the dates are seen to be very consistent, though the curve on which they lie is complicated.
- (d) Curve II. exhibits a well-marked hump and depression—slightly indicated in curve I.—which resembles the pause on the rise shown by the star's light curve.

9. In view of the doubt attached to certain parts of the curves, *e.g.* the gap in Baxendell's observations and the neighbourhood of E=50 to 57, it will be sufficient for the analysis to take the readings at twelve equidistant ordinates within a complete period of the inequality. We then get the figures in Table IV.

TABLE IV.
Ordinates of Curves I. and II.

E.	Curve I.	Curve II.	E.	Curve I.	Curve II.
3	0	- 2	33 $\frac{1}{2}$	+ 23	+ 30
8 $\frac{1}{2}$	- 32	- 32	38 $\frac{7}{12}$	+ 40	+ 34
13 $\frac{1}{8}$	- 64	- 63	43 $\frac{2}{8}$	+ 61	+ 60
18 $\frac{1}{4}$	- 60	- 60	48 $\frac{3}{4}$	+ 68	+ 71
23 $\frac{1}{8}$	- 31	- 32	53 $\frac{5}{8}$	+ 45	+ 43
28 $\frac{5}{8}$	- 2	- 1	58 $\frac{11}{8}$	+ 24	+ 22

Assuming these curves to be of the form—

$A + a \sin \theta + b \cos \theta + c \sin 2\theta + d \cos 2\theta + e \sin 3\theta + f \cos 3\theta$, etc.,
and analysing them harmonically we get:

Curve I. $A - 58.5 \sin \theta - 14.7 \cos \theta - 4.3 \sin 2\theta + 3.8 \cos 2\theta + 5.0 \sin 3\theta + 4.3 \cos 3\theta$.

Curve II. $A - 58.0 \sin \theta - 15.5 \cos \theta - 4.8 \sin 2\theta + 4.0 \cos 2\theta + 7.0 \sin 3\theta + 2.7 \cos 3\theta - 0.2 \sin 4\theta + 4.0 \cos 4\theta - 0.7 \sin 5\theta - 3.3 \cos 5\theta$.

These curves may be put in the form—

Curve I. $A + 60.3 \sin (\theta + 194^\circ.1) + 5.7 \sin (2\theta + 138^\circ.5) + 6.6 \sin (3\theta + 40^\circ.7)$.

Curve II. $A + 60.0 \sin (\theta + 195^\circ.0) + 6.1 \sin (2\theta + 140^\circ.2) + 7.5 \sin (3\theta + 21^\circ.1) + 4.0 \sin (4\theta + 87^\circ.1) + 3.4 \sin (5\theta + 258^\circ.0)$.

It will be seen that the first three harmonics in the two curves are very similar. The fourth and fifth harmonics are insignificant in curve I. and are omitted, but have a considerable amplitude in curve II.

10. We may now give the final solutions for the dates of maxima of S Herculis. Taking the means of columns 2 and 3 of Table IV. we get the values +6 and +5.7. We therefore add these to the constant at the head of column 4 of Table III., which becomes 2399266.8 and 2399266.5. Since the inequality runs through its course in 61 epochs, the argument will have the value 5°.9. The analyses above given are for epochs 3 to 64, and the value of 3 epochs expressed as an angle is 17°.7. Correcting, then, the curves to $E=0$ we have the following as the two formulæ for the star:—

$$\text{I. } 2399266^{\text{d}}.8 + 306^{\text{d}}.2 E + 60^{\text{d}}.3 \sin (5^{\circ}.9 E + 176^{\circ}.4) + 5^{\text{d}}.7 \sin (11^{\circ}.8 E + 103^{\circ}.1) + 6^{\text{d}}.6 \sin (17^{\circ}.7 E + 347^{\circ}.6).$$

$$\text{II. } 2399266^{\text{d}}.5 + 306^{\text{d}}.2 E + 60^{\text{d}}.0 \sin (5^{\circ}.9 E + 177^{\circ}.3) + 6^{\text{d}}.1 \sin (11^{\circ}.8 E + 104^{\circ}.8) + 7^{\text{d}}.5 \sin (17^{\circ}.7 E + 328^{\circ}.0) + 4^{\text{d}}.0 \sin (23^{\circ}.6 E + 16^{\circ}.3) + 3^{\text{d}}.4 \sin (29^{\circ}.5 E + 169^{\circ}.5).$$

11. As a test of the analysis it will suffice to take curve II. Comparing the dates in column 3 of Table III. with those com-

puted by the second formula, we obtain the residuals shown in Table V.

TABLE V.

E.	Adopted Date - Formula. d	E.	Adopted Date - Formula. d	E	Adopted Date - Formula. d
1	-5.2	33	+1.6	50	(-19.2)
2	(+1.2)	34	+4.8	51	-9.4
3	+1.2	37	(+6.3)	52	+9.4
6	-1.8	38	(+2.0)	58	+4.0
7	-3.6	39	-4.3	59	-3.6
8	-1.4	40	+0.8	60	+0.1
9	(+3.9)	41	(-4.1)	61	+3.4
10	(+1.5)	42	-0.4	62	+0.2
11	(-5.3)	43	+1.3	63	+6.6
12	-2.2	44	(+0.8)	64	+9.2
13	(+1.8)	45	(-1.3)	65	+0.8
26	(+3.9)	46	-5.7	66	-5.8
27	(+4.8)	47	-4.9	67	-4.2
29	-5.4	48	(-2.4)	68	-2.7
30	(+0.1)	49	+3.2	69	-1.0

It will be seen that with few exceptions these residuals are reasonably small, and probably within the limits of error in the deduced dates of maximum.

We may also compare the corrected periods found for the Baxendell, Rousdon, and B.A.A. series with those given for the same epochs by the formula (II.) This is done in Table VI.

TABLE VI.

E.	Periods.		O - F. d
	From Observations. d	From Formula. d	
1-7	300.6	301.0	-0.4
8-13	298.4	299.6	-1.2
26-34	312.5	312.3	+0.2
39-43	310.3	311.0	-0.7
44-49	307.6	308.1	-0.5
58-61	302.7	302.0	+0.7
62-65	300.5	301.2	-0.7
65-69	299.5	300.3	-0.8

Here, again, the residuals are satisfactory. It cannot be claimed, however, that the formula is definitive for the following reasons:—

- (a) Uncertainty as to the true mean value, which may be more or less than 306.2 days;
- (b) Doubt as to the homogeneity of the material used in the discussion, and as to the true form of certain parts of the curve;
- (c) The probability that, as is the case with the individual light curves, the curve is not exactly reproduced in successive periods of the inequality.

It may be remarked that the curve of dates should reach a minimum, *i.e.* the period of the star should be at its mean value a few years hence, and future observations will be of interest as a test of the work here tabulated.

12. Some acknowledgment must be made, finally, of the work of those who have provided the data for a discussion of this star.

It is a cause for great satisfaction that Professor Turner and Miss Blagg should be publishing in a form convenient for use the observations made so long ago by Baxendell, without which in some cases such discussions would be impossible.

Mr. Grover's persistent work at the Rousdon Observatory has not only carried on the observation of the star from the point where Baxendell left it, but its continuance to the present time adds enormously to its value; while the Variable Star Section of the British Astronomical Association, by combining the work of so many observers, is contributing material of inestimable worth to future investigators of stellar variation.

Ashted:
1915 May 8.

On the Short-Period Variable RR Lyrae. By C. Martin and H. C. Plummer. (Plates 20, 21.)

1. RR Lyrae is a variable of the cluster type discovered by Mrs. Fleming in 1901. Its designation in the Harvard Catalogue of variables is 192242, and it is identical with B.D. + 42° 3338 and with A.G.C. (Bonn) 12959. One hundred and eight photographic observations have been secured by Mr. Martin with the 15-inch reflector of this observatory between 1913 October and 1914 November. All the plates have been measured and reduced by him. The method of reduction has been sufficiently indicated in a series of papers on similar variables recently published by us in the *Monthly Notices*.

2. The magnitudes of the comparison stars have been based, as before, on two plates exposed under similar conditions both on the region of the variable and on the χ Persei cluster. The

magnitude scale of the plates is deduced from the stars in the latter region, for which we adopt the determinations of Schwarzschild. The results are given in the following table:—

TABLE I.

Star.	B.D.	Plate 1329. m_1 .	Plate 1373. m_2 .	Diff. $m_2 - m_1$.	Provisional Phot.-Mag.
<i>a</i>	+42°3325	7·30	7·33	+·03	7·31
<i>b</i>	+41°3352	7·73	7·78	+·05	7·76
<i>c</i>	+42°3340	8·32	8·39	+·07	8·36
<i>d</i>	+42°3331	8·97	9·02	+·05	9·00
<i>e</i>	+42°3336	9·18	9·21	+·03	9·19

Each exposure on the variable gives a set of residuals for the comparison stars. From an examination of these the following results are obtained:—

Star	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>
	m	m	m	m	m
Extreme range	·21	·21	·17	·16	·14
Probable error	±·030	±·025	±·024	±·023	±·014
Mean correction	-·018	+·014	-·017	-·003	-·001
Final magnitude	7·29	7·77	8·34	9·00	9·19

Here the extreme range is the sum of the largest positive and negative residuals in the series, each taken positively. The mean corrections are the means of each series of residuals, and when applied to the provisional magnitudes of Table I. give the final photographic magnitudes of the last line. These corrections are so small that it is unnecessary to revise the deduced magnitudes of the variable. They have, however, been applied to the residuals of the comparison stars, and the probable errors for a single exposure have been derived from the sum of each series thus corrected and taken without regard to sign.

3. The question of the epoch of maximum and the period is interesting. The available material may be arranged thus:—

Authority.	Epoch (T).	E.	$T - T_1$	$(T - T_1) - d \cdot 5668E.$
I. Wendell . . .	J.D. 2414856·500	...	...	...
II. Hertzsprung . .	18919·448	7168	4062·948	+·126
III. Lick . . .	19697·764	8541	4841·264	+·225
IV. Dunsink . . .	20398·371	9777	5541·871	+·267

Here I. and III. are epochs of visual maximum, II. and IV. of photographic maximum. The epoch I. is relatively so distant that a small error in it will be scarcely material. The period found by Kiess for the interval III.-I. is $0^d.566826$, which gives the residuals:

$$(II.-I.) - ^n.060, \quad (III.-I.) + ^d.003, \quad (IV.-I.) + ^d.013.$$

Kiess states in error that the discrepancy in II. is $0^d.115$, and infers that photographic maximum precedes visual maximum by 2.7 hours. But even the smaller effect of this kind is not confirmed by the Dunsink result. As a matter of fact, we are unaware of any parallel positive evidence of such an effect, though it is doubtless quite possible. Nevertheless the observed epochs cannot be reconciled with a single period. If we determine separate mean periods we obtain :

$$(II.-I.)^d.566818, \quad (III.-I.)^d.566826, \quad (IV.-I.)^d.566827, \\ (IV.-III.)^d.566834,$$

and we have a clear suggestion that the period is increasing. In particular, the last figure exceeds the period found by Kiess, whereas it should be in defect if the photographic precedes the visual maximum. We therefore seek to represent the epoch of maximum by the formula :

$$T = \text{J.D. } 241\,4856.500 + (.5668 + a)E + b(10^{-4}E)^2.$$

It is at once clear that all the dates cannot be reconciled, merely by noticing that the epoch III. is nearly the mean of II. and IV. Hence we shall bring together the visual dates I. and III. and the photographic dates II. and IV. The resulting equations for a and b are :

$$8541a + .729b = +.225$$

$$2609a + .442b = +.141,$$

which give

$$a = -.000002, \quad b = +1/3.$$

Other trials suggest that this simple value of b cannot be much improved on any supposition with the material at hand. The residuals now become :

$$(I.) .000, \quad (II.) -.031, \quad (III.) -.001, \quad (IV.) -.030.$$

The discrepancy between visual and photographic maximum is thus reduced to $0^d.03$, and it is quite impossible to be sure even of this amount. The result of plotting the visual observations suggests in both cases that the maximum may well have been placed too late, and the epoch IV. is certainly not too well defined by the observations.

Hence for the epoch of photographic maximum we adopt

$$T = \text{J.D. } 241\,4856.470 + ^d.566798E + (10^{-4}E)^2 \div 3,$$

corresponding to which the period is

$$P = 0^d.566798 + .000067(10^{-4}E).$$

The periods at the successive epochs are respectively :

$$(I.)^d.566798, \quad (II.)^d.566846, \quad (III.)^d.566855, \quad (IV.)^d.566863.$$

Wendell * found the period $0^d.5668$, which is confirmed for the date (1899) to a higher order of accuracy than that recorded. The spectrographic observations of Kiess † extend over 800 periods, so that the difference from his adopted period ($0^d.00003$) should have a slight but sensible effect. The last period is confirmed by the internal evidence of the Dunsink observations. The early exposures 6 and 7 belong to the steep ascending branch of the magnitude curve. Reduced with the period $0^d.566826$, they showed a manifest discordance which is entirely removed by the period now adopted.

4. The details of the observations of RR Lyræ are given in Table II. The second column gives the date and G.M.T. of the middle of the exposure. No reduction to heliocentric time has been applied. The next column gives the corresponding Julian date with decimal fraction of a day from G.M. noon. The phase in the fourth column is expressed as a decimal fraction of a day, and is reckoned from the preceding epoch of maximum, based on the formula :

$$T = \text{J.D. } 2420398.371 + 0^d.566863E.$$

TABLE II.

Serial No.	Date G. M. T.		Julian Day.	Phase in days.	Exposure.	Magn.	Obs. —Curve.	Images.
	1913.	h m	2420		min.		m	
1	Oct. 31	7 27	072.311	.453	20	8.67	+ .01	Fair.
2		31 7 47	072.325	.467	20	8.69	+ .02	Fair.
3	Nov. 21	6 7	093.254	.423	20	8.50	— .06	Fair.
4		21 6 27	093.269	.438	20	8.58	— .02	Fair.
5		21 6 47	093.283	.452	20	8.60	— .05	Fair.
6		21 8 38	093.360	.529	20	8.09	— .03	Good.
7		21 8 58	093.374	.543	20	7.97	— .01	Good.
8		26 8 24	098.350	.416	20	8.51	— .04	Fair.
9	Dec. 1	8 24	103.350	.315	20	8.52	— .01	Fair.
10		1 8 44	103.365	.330	20	8.57	— .02	Bad.
	1914.							
11	June 16	12 16	300.511	.207	20	8.42	— .02	Fair.
12	July 28	12 24	342.517	.265	20	8.47	— .04	Fair.
13		28 12 44	342.532	.280	20	8.50	— .04	Fair.
14		30 11 55	344.496	.544	20	8.03	+ .05	Fair.
15		30 12 16	344.511	.559	20	7.92	+ .04	Fair.
16	Aug. 12	9 55	357.413	.423	20	8.55	— .01	Good.
17		12 10 23	357.433	.443	20	8.60	— .02	Fair.
18		15 13 4	360.545	.154	20	8.20	— .03	Fair.
19		15 13 25	360.560	.169	20	8.25	— .03	Fair.

* *Harvard Annals*, lxix, part 1.† *L. O. B.*, No. 232.

TABLE II.—*continued.*

Serial No.	Date G.M.T.		Julian Day.	Phase in days.	Exposure.	Magn.	Obs.—Curve.	Images.
	1914.	h m	2420		min.		m	
20	Aug.	15 13 53	360'579	'188	20	8'35	'00	Fair.
21		15 14 11	360'590	'199	15	8'34	— '06	Fair.
22		17 9 48	362'409	'317	20	8'53	— '01	Good.
23		17 10 8	362'422	'330	20	8'55	— '03	Fair.
24		17 10 40	362'445	'353	20	8'58	— '03	Good.
25		17 11 1	362'459	'367	20	8'56	— '02	Good.
26		17 11 27	362'477	'385	20	8'52	— '03	Fair.
27		17 11 50	362'493	'401	20	8'58	+ '03	Good.
28		17 12 17	362'512	'420	20	8'63	+ '08	Good.
29		17 12 37	362'526	'434	20	8'58	— '01	Good.
30		18 9 49	363'409	'184	20	8'37	+ '04	Fair.
31		18 10 19	363'431	'206	20	8'40	— '03	Good.
32		18 10 39	363'445	'220	20	8'44	— '02	Fair.
33		18 11 53	363'495	'270	20	8'53	+ '01	Fair.
34		18 12 14	363'510	'285	20	8'57	+ '03	Good.
35		21 9 43	366'406	'346	20	8'61	'00	Good.
36		21 10 4	366'420	'360	20	8'56	— '03	Fair.
37		21 10 30	366'438	'378	20	8'61	+ '05	Fair.
38		21 10 51	366'453	'393	20	8'50	— '05	Fair.
39		21 11 18	366'471	'411	20	8'54	— '01	Fair.
40		21 11 39	366'486	'426	20	8'57	+ '01	Fair.
41		24 11 2	369'459	'565	20	7'84	— '03	Good.
42		24 11 23	369'474	'013	20	7'88	— '03	Fair.
43		24 11 49	369'492	'031	20	8'02	+ '02	Fair.
44		24 12 48	369'534	'073	20	8'26	— '01	Fair.
45		24 13 7	369'546	'085	15	8'28	— '04	Fair.
46		26 10 39	371'445	'283	19	8'56	+ '02	Fair.
47		31 10 24	376'434	'171	16	8'33	+ '04	Fair.
48		31 10 47	376'450	'187	16	8'40	+ '06	Fair.
49	Sept.	10 8 43	386'364	'464	16	8'65	— '02	Good.
50		11 8 39	387'361	'327	16	8'59	+ '02	Fair.
51		11 8 56	387'372	'338	16	8'60	— '01	Fair.
52		11 9 22	387'390	'356	16	8'58	— '02	Fair.
53		11 9 45	387'407	'373	16	8'58	+ '01	Fair.
54		11 10 9	387'423	'389	16	8'52	— '03	Fair.
55		11 11 27	387'477	'443	16	8'64	+ '02	Fair.
56		14 8 30	390'354	'486	16	8'51	— '03	Fair.
57		14 8 47	390'366	'498	16	8'41	— '01	Fair.

TABLE II.—*continued.*

Serial No.	Date G.M.T.			Julian Day.	Phase in days.	Ex- posure.	Magn.	Obs. —Curve.	Images.
	1914.	h	m						
	Sept.	14	9 10	2420 390'382		min.		m	
58		14	9 10	390'382	'514	16	8'31	+ '04	Fair.
59		14	9 26	390'393	'525	16	8'20	+ '04	Fair.
60		14	9 50	390'410	'542	16	8'01	+ '01	Fair.
61		14	10 6	390'421	'553	16	7'96	+ '05	Fair.
62		15	8 46	391'365	'363	20	8'61	+ '02	Fair.
63		15	9 6	391'379	'377	20	8'62	+ '06	Fair.
64		15	9 34	391'399	'397	20	8'60	+ '05	Fair.
65		15	9 56	391'414	'412	20	8'59	+ '04	Fair.
66		15	10 30	391'438	'436	20	8'58	- '02	Fair.
67		15	10 51	391'453	'451	20	8'68	+ '03	Bad.
68		15	11 18	391'471	'469	20	8'65	- '01	Fair.
69		15	11 38	391'485	'483	20	8'65	+ '07	Fair.
70		16	9 34	392'399	'263	20	8'47	- '04	Good.
71		16	9 55	392'413	'277	20	8'55	+ '02	Fair.
72		16	10 24	392'434	'298	22	8'58	+ '05	Good.
73		16	10 46	392'449	'313	20	8'56	+ '03	Fair.
74		17	9 6	393'379	'110	24	8'18	- '06	Fair.
75		17	9 31	393'396	'127	24	8'10	- '09	Fair.
76		17	10 10	393'424	'155	24	8'28	+ '05	Fair.
77		17	10 35	393'442	'173	24	8'33	+ '04	Fair.
78		17	11 12	393'466	'197	20	8'39	'00	Fair.
79		21	8 7	397'337	'100	24	8'33	+ '05	Fair.
80		21	8 32	397'355	'118	24	8'20	'00	Fair.
81		21	9 6	397'379	'142	24	8'23	+ '03	Fair.
82		21	9 31	397'396	'159	24	8'23	- '02	Good.
83		21	10 6	397'421	'184	27	8'35	+ '02	Fair.
84		21	10 32	397'439	'202	24	8'38	- '04	Fair.
85		21	11 9	397'464	'227	24	8'48	+ '02	Fair.
86		21	11 34	397'482	'245	24	8'43	- '03	Fair.
87		22	8 41	398'362	'558	24	7'88	- '01	Fair.
88		22	9 5	398'379	'008	24	7'88	'00	Good.
89		29	8 17	405'345	'172	20	8'31	+ '02	Good.
90		29	8 44	405'365	'192	20	8'33	- '03	Fair.
91		29	9 39	405'403	'230	20	8'45	- '01	Good.
92		29	10 8	405'422	'249	20	8'49	+ '02	Good.
93	Oct.	19	8 22	425'348	'334	24	8'55	- '05	Fair.
94		22	9 33	428'398	'550	20	7'90	- '02	Fair.
95		22	9 54	428'413	'565	20	7'82	- '05	Fair.

TABLE II.—*continued.*

Serial No.	Date G.M.T.		Julian Day.	Phase in days.	Exposure.	Magn.	Obs.—Curve.	Images.
	1914.	h m						
			2420		min.		m	
96	Oct. 29	6 55	435'288	'071	20	8'25	— '01	Fair.
97	29	7 21	435'306	'089	20	8'29	— '03	Fair.
98	29	8 11	435'340	'123	20	8'17	— '02	Fair.
99	Nov. 6	7 38	443'319	'166	20	8'36	+ '09	Good.
100		6 8 5	443'337	'184	20	8'35	+ '02	Fair.
101		6 8 38	443'360	'207	20	8'44	'00	Fair.
102		6 9 22	443'390	'237	20	8'47	+ '01	Good.
103		11 6 49	448'284	'029	20	8'02	+ '03	Good.
104		11 7 9	448'298	'043	20	8'08	+ '01	Good.
105		16 6 10	453'257	'467	20	8'69	+ '02	Fair.
106		16 7 19	453'306	'516	24	8'19	— '05	Fair.
107		16 7 41	453'321	'531	20	8'01	— '09	Fair.
108		16 8 2	453'334	'544	20	7'89	— '08	Fair.

The time of exposure is given in the fifth column, and the concluded photographic magnitude of the variable in the sixth column. The quality of the images on the plates is indicated in the last column.

The penultimate column of Table II. shows the deviations of the observed magnitudes from a certain curve represented in fig. 1 (Plate 20). This curve is based on the formula :

$$\begin{aligned}
 m = & 8.36 + .27 \sin(\theta + 234^\circ) + .10 \sin(2\theta + 260^\circ) \\
 & + .11 \sin(3\theta + 286^\circ) + .08 \sin(4\theta + 287^\circ) \\
 & + .02 \sin(6\theta + 131^\circ) + .02 \sin(9\theta + 305^\circ),
 \end{aligned}$$

which results from the harmonic analysis of a curve drawn by graphic interpolation among the observations, terms with coefficients less than .015 being neglected. The angle θ runs from the maximum and completes its period 360° in the period of the star. The curve appears to be a relatively simple one among its class. The subsidiary maximum near the phase 0^d.40 just before the true minimum, and also that which occurs about half a period earlier, are believed to be real. The intermediate sinuosities of the curve are merely the result of attempting to represent it by the sum of a few harmonic terms. The average discordance of the observations from the curve is $\pm .03$, which is of an order consistent with the results obtained for the comparison stars.

5. In fig. 2 (Plate 21) are represented five curves relating to RR Lyræ. These are:—

A. The radial velocity curve based on the spectrographic observations of Kiess. According to the usual convention, negative velocities indicate approach to the Sun.

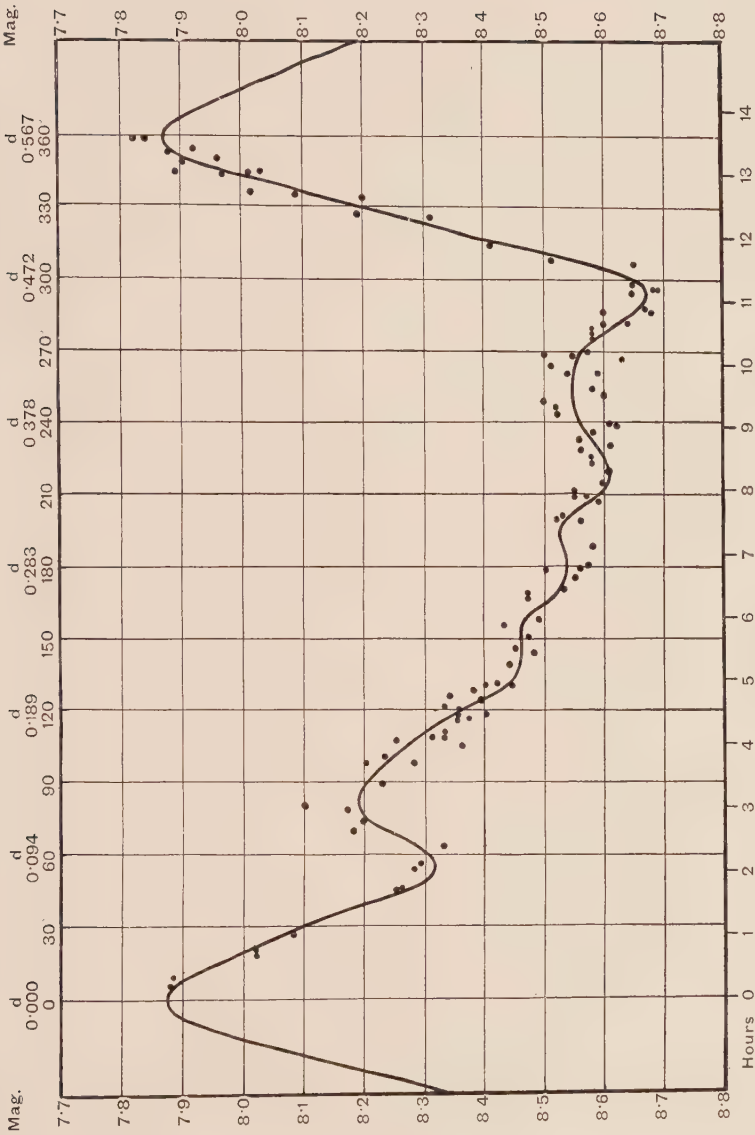


Fig. 1.—Photographic Light Curve of RR Lyræ.

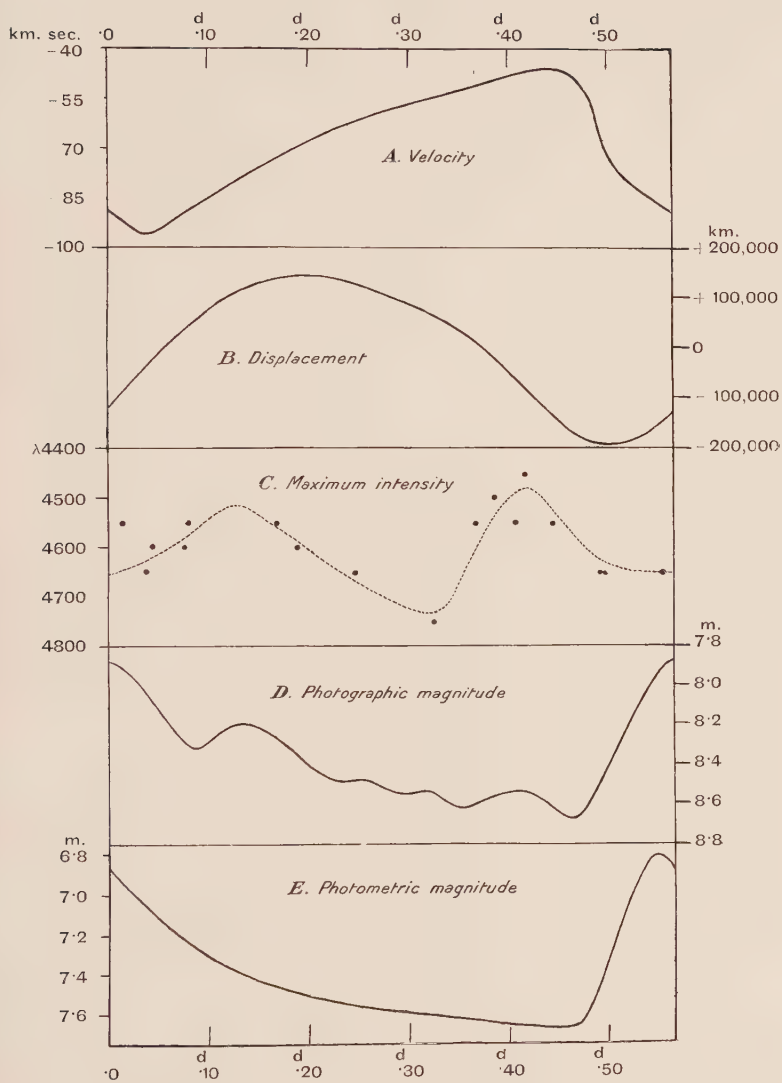


Fig. 2.—Curves relating to RR Lyræ.

B. What is here called the displacement curve, related to the curve *A* in a way which will be described.

C. The curve which indicates the position of maximum intensity in the photograph of the spectrum, based on the measurement of seventeen plates by Kiess.

D. The photographic magnitude curve, which differs from the curve in fig. 1, plate 20, only in scale.

E. The photometric magnitude curve, based on the measures of Wendell. This curve is based on the epoch adopted by Wendell, and a slightly earlier position of the maximum, as shown, seems consistent with the observations. But no stress is laid on this point.

It is usual to find a greater range between maximum and minimum in the photographic than in the visual magnitude curve. This does not seem to be the case here. The range in the former is about $0^m.80$, and in the latter slightly more. According to Kiess the visual range is even greater. With two exceptions the curves *D* and *E* appear strikingly similar. The points of difference lie in the two subsidiary maxima in the photographic curve to which attention has been drawn, and of which there is no indication in the visual curve. In this connection the curve *C* of greatest spectral intensity is of great interest, for it shows that this point of the spectrum moves towards the blue end at the corresponding phases in the light curves, and thus explains the discrepancies between the visual and photographic results. The curve *C* is somewhat conjectural, being based on no more than seventeen plates, as indicated, and the interpretation of results of this kind is not altogether free from uncertainty. But the inference that the photosphere of the star becomes bluer (presumably owing to rising temperature) at these two points in the period seems beyond doubt. We have noticed before similar irregularities in the photographic light curves of stars of this class, especially the minor maximum at about seven-tenths of the period after the main maximum (notably in the case of SW Draconis). Though these features are not reproduced in the visual curves, we believed them to be real. The evidence of the distribution of intensity in the continuous spectrum adds weight to this belief.

6. The natural interpretation of the velocity curve *A* is that it represents the radial component of the velocity of one member of a binary system. The corresponding orbit on this hypothesis has been deduced by Kiess. But although this is the most facile explanation of the periodic displacement of the spectral lines, it is very far from being free from difficulty in some cases. Accordingly we have suspected that this is not the true explanation, and have hinted * that such displacements should be attributed as a whole or in part to radial pulsations in the atmosphere of the star. More recently this idea has been put forward by Mr. Shapley † as applying to all stars of δ Cephei type. When the

* *M.N.*, lxxiii. p. 665, lxxiv. p. 662.

† *Ap. J.*, xl. p. 448.

period amounts to several days this implies oscillations on an enormous scale. If such can be entertained, there is even less difficulty with the shorter periods of the cluster type stars. Now the velocity V can be represented by the formula:

$$-V = 69 + 21 \sin(\theta + 44^\circ) + 7 \sin(2\theta + 69^\circ) + 2 \sin(3\theta + 126^\circ)$$

with smaller terms which become even less important on integration. Hence the corresponding periodic displacement measured from the centre of the star is

$$D = - \int (V + 69) dt \\ = - \{ 21 \cos(\theta + 44^\circ) + 4 \cos(2\theta + 69^\circ) + 1 \cos(3\theta + 126^\circ) \} P / 2\pi,$$

where P is the period of the star in seconds. Thus

$$D = 1000 \{ 163 \sin(\theta + 314^\circ) + 29 \sin(2\theta + 339^\circ) + 6 \sin(3\theta + 126^\circ) \}$$

is on this interpretation the radial displacement, expressed in kilometres, of the star's surface. This formula is represented by the curve B in fig. 2, Plate 21. Since the spectrum corresponds to the integrated light of one hemisphere, the centre of the lines should be less displaced than by the amount shown by the centre of the disc, and the lines should be more diffuse on the side of less displacement. But such effects would be diminished by any considerable darkening towards the limb.

7. The cycle of changes which take place in the star on this hypothesis is easily traced from a comparison of the curves given. At the time when the star is brightest the absorbing layer is expanding, having reached its lowest level some 100 minutes earlier. At this point observation of the spectrum proves that absorption is greatly diminished, and this is the chief factor in bringing about the maximum. The level of the surface is about 120,000 km. below the mean (in respect to time). From this point expansion continues for about five hours until a level of 140,000 km. above the mean is reached. In the interval absorption increases and the visual brightness declines steadily. For some reason, within this interval the star becomes bluer and the decrease in the actinic light is temporarily checked and even reversed. During the next seven hours, or roughly half the period, the star steadily contracts. At first the star becomes redder and very slightly less bright. But again before the minimum is reached the colour changes towards blue and the decline in the photographic magnitude is briefly arrested. Minimum light then occurs, apparently before the star has contracted to its lowest level, about 200,000 km. below the mean. Expansion then begins again, and not long after the radiation of the star is once more at its maximum.

8. A theory of this kind cannot be tested by a single example, but even one case examined in detail serves to illustrate the operations involved. The effects expressed in the magnitude curve are exceedingly complicated and depend on several factors

which at different stages may reinforce or compensate one another. The factors mainly concerned may be taken to be the varying size of the star, the radiating power of the photosphere, and absorption in the outer layer. In general such stars appear to be bluer at maximum and redder at minimum, and the range is correspondingly greater in the photographic light curve. But in the case of RR Lyrae the distribution of energy in the spectrum remains unchanged from minimum to maximum, and thus the equal range in the two curves is accounted for. When a corresponding theory is extended to the stars of δ Cephei type, pulsations on an enormous scale must be contemplated, owing to the longer periods. Since, however, there is good reason to believe that these stars are of great size and low density, the pulsations may nevertheless be on a similar scale relatively to the diameters. According to Hertzsprung,* RR Lyrae possesses a centennial proper motion of about $25''$, and therefore it resembles in absolute brightness (and presumably density) the Sun rather than the giant variable stars.

9. Little consideration seems needed to understand that pulsations of this kind are possible in a star. Let us imagine a star constituted of a perfect gas, with k the specific heat at constant volume and γk the specific heat at constant pressure, so that

$$p = k(\gamma - 1)\rho t.$$

The intrinsic heat energy of a shell between the radii r and $r + dr$ is

$$dE = k\rho t \cdot 4\pi r^2 dr$$

or

$$(\gamma - 1)dE = 4\pi \cdot p r^2 dr.$$

Again, if m is the total mass inside the shell, equilibrium requires that

$$\frac{dp}{dr} = -\alpha\rho\frac{m}{r^2}$$

and the work done by m in bringing the material of the shell into position from a state of infinite diffusion is

$$dW = \alpha\rho m \cdot 4\pi r dr = -4\pi r^3 dp.$$

Hence

$$3(\gamma - 1)dE - dW = 4\pi(3pr^2 dr + r^3 dp) = 4\pi d(pr^3),$$

or integrating from the centre to any point

$$3(\gamma - 1)E - W = 4\pi pr^3,$$

which, apart from any assumption as to the form of equilibrium, convective or otherwise, expresses the pressure at any point in terms of the interior heat energy and loss of gravitational energy.

At the free surface of a star $p = 0$ and

$$3(\gamma - 1)E = W,$$

* *A. N.*, 4692.

an equation found by Perry.* If $\gamma \leq 1\frac{1}{5}$ the distribution in convective equilibrium is infinite and the equation scarcely applies. If $\gamma > 1\frac{1}{5}$, $E < W$ and the balance of energy may be imagined to be spent in radiation. When $1\frac{1}{5} < \gamma < 1\frac{1}{3}$ a mass of gas can exist in convective equilibrium in the form of a finite sphere only on the condition that it is supplied with greater energy than it can possibly obtain by its own contraction under gravity. Hence Perry was led to suggest that no substance for which $\gamma \leq 1\frac{1}{3}$ can behave as a perfect gas. It seems at least equally logical to assert that for such a substance spherical equilibrium is out of the question. This remarkably simple proposition may then find wide application in a number of ways which will naturally suggest themselves.

It is sufficiently obvious that the history of the development of a star is that of a continuous struggle on the part of the gravitational forces to readjust the material in such a way as to compensate the loss of energy by radiation, a process complicated by such factors as the rotation of the body. The bearing of the very definite theorem just stated is to suggest that the necessary adjustment is by no means so easily effected as might at first sight be imagined, and this inference is not really affected by the fact that the substances of nature probably do not possess the ideally simple properties involved in the argument. From this point of view it may well be that the conditions unfavourable to a stable arrangement present themselves particularly at certain stages of development and density. It is in harmony with this idea (as Shapley has also remarked) that the variations of the cluster type and of the δ Cephei type, although undoubtedly very similar in nature, tend to the two distinct periods 12 hours and 7 days respectively. On the other hand, the frequency distribution of the periods is much less easy to understand on any theory involving a binary system. Although the stars of a cluster are certainly not of one spectral type, they may be expected to approximate to a predominant stage of development, and hence the occurrence of many variables of one type in certain clusters and their absence in others finds some explanation. Apparently clusters have not been found which contain variables of the δ Cephei type. At the same time stars of the latter class show a distinct condensation towards the galactic plane. There are therefore points of difference between the two classes, and it is likely that the two types should not be regarded as two phases in the history of the same star. But in any case hardly any theory can be more unpromising than one which assumes the existence of close binary systems presenting all the conditions necessary to produce eclipse effects, when such effects are invariably absent.

* *Nature*, lx. p. 252.

The Effect of a Resisting Medium on Lagrange's Three Particles.

By Harold Jeffreys, B.A., M.Sc.

In general the effect of a resisting medium on a system executing small oscillations about a steady motion is to cause these to die down. On the other hand, it is stated by Professor See* that in the problem of three bodies the effect of the resisting medium is to cause the constant of the Jacobi integral to increase with the time, which is probably true in the case of an ordinary satellite. If, however, C increases in the case of a Trojan planet, then as $\Omega > \frac{1}{2}C$ and the third point of the equilateral triangle is a minimum point for Ω , it follows that the displacement of the small body from its mean place must increase with the time, so that it is secularly unstable. It therefore becomes interesting to examine which alternative is correct.

The motion of the small body is found to be governed by equations of motion of the type discussed in the next article, and that its oscillations actually do die down. C therefore diminishes secularly in this case. The resisting medium cannot then cause a Trojan planet to depart from its mean position to such an extent that it may become a minor planet or a satellite.

THE INFLUENCE OF VARIABLE DISSIPATIVE FORCES IN
A GYROSCOPIC SYSTEM.*Definition of the System.*

Let the modified Lagrangian function, expanded as far as the second order in the variations of the co-ordinates, be given by

$$\begin{aligned} 2R = & a_{11}\dot{q}_1^2 + a_{22}\dot{q}_2^2 + \dots + a_{nn}\dot{q}_n^2 \\ & + 2a_{12}\dot{q}_1\dot{q}_2 + \dots + 2a_{n-1,n}\dot{q}_{n-1}\dot{q}_n \\ & + 2(c_{12} + d_{12})q_1\dot{q}_2 + 2(c_{21} - d_{21})q_2\dot{q}_1 + \dots \\ & + b_{11}q_1^2 + b_{22}q_2^2 + \dots + b_{nn}q_n^2 \\ & + 2b_{12}q_1q_2 + \dots + 2b_{n-1,n}q_{n-1}q_n. \end{aligned}$$

Also let the dissipation function be given by

$$\begin{aligned} 2F = & e_{11}\dot{q}_1^2 + e_{22}\dot{q}_2^2 + \dots + e_{nn}\dot{q}_n^2 \\ & + 2e_{12}\dot{q}_1\dot{q}_2 + \dots + 2e_{n-1,n}\dot{q}_{n-1}\dot{q}_n \\ & + 2f_{11}q_1\dot{q}_1 + \dots + 2f_{nn}q_n\dot{q}_n \\ & + 2(f_{12} + g_{12})q_1\dot{q}_2 + 2(f_{21} - g_{21})q_2\dot{q}_1 \end{aligned}$$

where $a_{rs} = a_{sr}$, $c_{rs} = c_{sr}$, $d_{rs} = d_{sr}$, $b_{rs} = b_{sr}$, $e_{rs} = e_{sr}$, $f_{rs} = f_{sr}$, $g_{rs} = g_{sr}$. Friction is assumed not to influence the ignorable co-ordinates.

Then the quantities a , e , d , b are supposed constant. The quantities e , f , g are small quantities of the first order, and their

* *Capture Theory*, p. 222.

rates of change, and also $\frac{d^2e}{dt^2}$, $\frac{d^2f}{dt^2}$, and $\frac{d^2g}{dt^2}$, are supposed to be small quantities of the second order.

The Equations of Motion.

We obtain then n equations of the type

$$\begin{aligned} \{a_{11}\ddot{q}_1 + e_{11}\dot{q}_1 + (b_{11} + f_{11})q_1\} + \{a_{12}\ddot{q}_2 - 2d_{12}\dot{q}_2 + e_{12}\dot{q}_2 \\ + (b_{12} + f_{12} - g_{12})q_2\} + \dots + \dots = 0 \\ \{a_{21}\ddot{q}_1 + 2d_{21}\dot{q}_1 + e_{21}\dot{q}_1 + (b_{21} + f_{21} + g_{21})q_1\} + \{a_{22}\ddot{q}_2 + e_{22}\dot{q}_2 + \\ (b_{22} + f_{22})q_2\} + \dots + \dots = 0 \end{aligned}$$

etc., etc.

To solve these, assume as a first approximation $q_1 = \lambda_1 e^{\gamma t}$, $q_2 = \lambda_2 e^{\gamma t}$, and so on, where γ , λ_1 , λ_2 , etc., are constants. This gives n homogeneous linear equations to find the ratios of the quantities λ .

We then get a determinantal equation for γ , of the $2n$ 'th degree with real coefficients. Let the determinant be $\Delta(\gamma)$. At present neglect the variation of e , f , and g .

The imaginary roots must occur in conjugate pairs. Let one of these pairs be $\gamma_1 = \alpha + i\beta$; $\gamma_2 = \alpha - i\beta$.

If in $\Delta(\gamma)$ we change e to $-e$, g to $-g$, and γ to $-\gamma$ throughout in the elements, the determinant is unaltered. It follows that α is an odd function of the e 's and the g 's, while β is either odd or even. β is, however, not a small quantity, and its sign therefore cannot be determined by the small quantities e and g . Hence β is an even function of e and g .

Such a pair of roots having been found, consider one of them as substituted into the equations connecting the quantities λ . Any $n - 1$ of these equations may then in general be solved to obtain the ratios of the λ 's. A suitable solution has therefore been found. It will hold with sufficient accuracy until the variations of e , f , and g have caused γ and λ to be appreciably modified.

If now we include the variations of e , f , and g , we can still solve the equations at any instant, but γ and λ , instead of being constants, are now known functions of the time. The total variation of any of them, however, must always be a small quantity, as it is due to the variation of e , f , and g , while their rates of variation are small quantities of the second order, as they contain $\frac{de}{dt}$, $\frac{df}{dt}$, and $\frac{dg}{dt}$ as factors.

Try, then, as a second approximation

$$q_1 = \mu_1 \exp \int (\gamma_0 + \delta\gamma) dt,$$

$$q_2 = \mu_2 \exp \int (\gamma_0 + \delta\gamma) dt, \text{ etc.}$$

where each μ differs slightly from the corresponding λ , and γ_0 is one of the values of γ already found. $\frac{d\mu}{dt}$ and $\frac{d^2\mu}{dt^2}$ are small quantities of the second order at least.

$$\begin{aligned} \dot{q}_1 &= \left\{ (\gamma_0 + \delta\gamma)\mu_1 + \frac{d\mu_1}{dt} \right\} \exp \int (\gamma_0 + \delta\gamma) dt, \\ \ddot{q}_1 &= \left\{ (\gamma_0 + \delta\gamma)^2\mu_1 + 2(\gamma_0 + \delta\gamma)\frac{d\mu_1}{dt} + \left(\frac{d\gamma_0}{dt} + \frac{d}{dt}\delta\gamma \right)\mu_1 + \frac{d^2\mu_1}{dt^2} \right\} \\ &\quad \times \exp \int (\gamma_0 + \delta\gamma) dt. \end{aligned}$$

Now substituting in the equations of motion we get, provided none of the μ 's is 0,

$$\Delta(\gamma_0 + \delta\gamma) + h = 0$$

where h is a small quantity of the second order.

But $\Delta(\gamma_0) = 0$, and therefore

$$\delta\gamma = - \frac{h}{\left(\frac{\partial \Delta}{\partial \gamma} \right)_{\gamma=\gamma_0}} = \text{a small quantity of the second order.}$$

Hence $\int \delta\gamma dt = \text{a small quantity of the first order, provided } \gamma_0$ is not a multiple root of $\Delta = 0$, so that if we neglect small quantities in the final values of the co-ordinates, the system

$$\begin{aligned} q_1 &= \lambda_1 \exp \int \gamma_0 dt, \\ q_2 &= \lambda_2 \exp \int \gamma_0 dt, \text{ etc.} \end{aligned}$$

is an adequate approximation for all values of the time.

If one of the μ 's, say μ_r , is 0, then the q_r portion of each equation is a small quantity of the second order; but the other μ 's may be eliminated as before, and $\delta\gamma$ is again a small quantity of the second order. In this case it no longer holds that $q_r \propto \exp \int \gamma_0 dt$ with sufficient accuracy. All we can say is that q_r is small.

It is necessary to the above argument that $\frac{de}{dt}$, $\frac{df}{dt}$, and $\frac{dg}{dt}$ be of the second order. If they are only of the first order, h may be of the first order, and we cannot be sure that the neglect of $\int \delta\gamma dt$ is justified.

It follows, then, that the amplitude of vibration in each considerable co-ordinate is nearly proportional to $\exp \int a dt$ for all time.

In certain cases it happens that $e_{11} = \kappa a_{11}$, $e_{12} = \kappa a_{12}$, etc., $g_{12} = \kappa d_{12}$, etc., and all the f 's are zero.

In such a case, to solve the algebraic equation $\Delta(\gamma) = 0$ try as a solution $\gamma = m - \frac{\kappa}{2}$. Then the typical element of the determinant, namely $a\gamma^2 + 2d\gamma + e\gamma + b + f + g$, becomes $am^2 + 2dm + b + a$ second order quantity.

Then

$$\left\{ \Delta\left(m - \frac{\kappa}{2}\right) \right\}_{e=\kappa a, g=\kappa d} = \left\{ \Delta(m) \right\}_{e=g=0} + \text{a second order quantity.}$$

If then m satisfies the determinantal equation when the dissipative forces are neglected, we have $\left\{ \Delta(m) \right\}_{e=g=0} = 0$, and therefore $\left\{ \Delta\left(m - \frac{\kappa}{2}\right) \right\}_{e=\kappa a, g=\kappa d}$ is a small quantity of the second order.

$\gamma = m - \frac{\kappa}{2}$ is then a sufficiently accurate solution for the purpose in question, provided m is a simple root of $\left\{ \Delta(m) \right\}_{e=g=0} = 0$. im is then the speed of a free vibration where dissipative forces are neglected.

Provided then that m is purely imaginary, we see that $\alpha = -\frac{\kappa}{2}$; so that if the system is stable, and is secularly stable for one degree of freedom, it is secularly stable for all degrees of freedom.

The above method is capable of describing the history of a particular free oscillation whether the system as a whole be stable or not, as each purely imaginary root of $\Delta(m) = 0$ has been considered separately. If real or complex roots of $\Delta(m) = 0$ do arise, however, the system has not ordinary stability and therefore the question of secular stability does not concern us.

Application to Lagrange's Three Particles.

Now consider the case of Lagrange's three particles, supposed to be moving in a resisting medium. The particles S and J are regarded as large, while M is too small to affect the motion of J. Using then astronomical units of mass, the equations of motion of J relative to S are

$$\ddot{X} = -(S+J)\frac{X}{R^3} - L\dot{X},$$

$$\ddot{Y} = -(S+J)\frac{Y}{R^3} - L\dot{Y},$$

$$\ddot{Z} = -(S+J)\frac{Z}{R^3} - L\dot{Z}$$

where XYZ are the co-ordinates of J relative to S, referred to axes fixed in direction. L is small and essentially positive. $R = (X^2 + Y^2 + Z^2)^{\frac{1}{2}}$.

If $L=0$, a particular solution is $X = A \cos nt$, $Y = A \sin nt$, $Z = 0$ where $n^2 A^3 = S + J$.

To the second approximation $Z=0$ is still a solution. Put then $\tan^{-1} Y/X = w$, and $L = KR^{-m}$.

$$\text{Then} \quad \frac{d}{dt}(R^2 \dot{w}) = -L(X\dot{Y} - \dot{X}Y) = -LnA^2$$

$$\therefore \quad \frac{d}{dt}(R^2 \dot{w}) = -LnA^2.$$

$$\text{Also} \quad \ddot{R} - R\dot{w}^2 = -\frac{S+J}{R^2} - L\dot{R}.$$

The last term is of the second order, and can be omitted.

An approximation to a solution is then

$$R^m - R_0^m = -2mK(t - t_0)$$

where R_0 , t_0 refer to the initial conditions.

$$\frac{dw}{dt} = n = n_0 \left(\frac{R_0}{R} \right)^{\frac{3}{2}}.$$

This gives then the variation of Jupiter's circular orbit with the time.

To find the motion of M, take the origin O to be such that the angle JSO = $\pi/3$ and SO = R, and let the production of SO be the axis of x . The axes are then rotating with angular velocity n . The gravitational accelerations of M relative to these axes are then respectively,

$$\left. \begin{aligned} & -\frac{J}{4R^3}(x - 3\sqrt{3}y) + 2\frac{S}{R^3}x \\ & \frac{J}{4R^3}(3\sqrt{3}x + 5y) - \frac{S}{R^3}y \\ & -\frac{S+J}{R^3}z \end{aligned} \right\}.$$

The resistance terms are proportional to the excess of the velocity of M over the velocity of the medium at the point. They are then of the form $k(\dot{x} - yn - u)R^{-m}$, $k(\dot{y} + xn + nR - v)R^{-m}$, and $k\dot{z}R^{-m}$. k/K is great, since the resisting force varies as the square of the diameter of the body, while the mass varies as the cube of the diameter. Hence the acceleration is inversely as the diameter. k is nevertheless small absolutely.

The equations of motion are then

$$\begin{aligned}\ddot{x} - 2n\dot{y} - y\dot{n} - xn^2 &= -\frac{J}{4R^3}(x - 3\sqrt{3}y) + \frac{2S}{R^3}x - k(\dot{x} - yn - u)R^{-m} \\ \ddot{y} + 2n\dot{x} + x\dot{n} - yn^2 &= \frac{J}{4R^3}(3\sqrt{3}x + 5y) - \frac{S}{R^3}y - k(\dot{y} + xn + nR - v)R^{-m} \\ \ddot{z} &= -\frac{S+J}{R^3}z - k\dot{z}R^{-m}.\end{aligned}$$

Substituting for n^2 we find

$$\begin{aligned}\ddot{x} - 2n\dot{y} - y\dot{n} &= ax + hy - k(\dot{x} - ny - u)R^{-m} \\ \ddot{y} + 2n\dot{x} + x\dot{n} &= hx + by - k(\dot{y} + nx + nR - v)R^{-m} \\ \ddot{z} &= -n^2z - k\dot{z}R^{-m}\end{aligned}$$

where

$$\begin{aligned}a &= \frac{3(S + \frac{1}{4}J)}{S + J}n^2 \\ h &= \frac{3\sqrt{3}}{4} \frac{J}{S + J}n^2 \\ b &= \frac{9}{4} \frac{J}{S + J}n^2.\end{aligned}$$

Now R and n are slowly varying quantities. Take $w = \int ndt$ as independent variable. It is found that

$$\begin{aligned}\frac{d^2x}{dw^2} + \kappa \frac{dx}{dw} - 2\frac{dy}{dw} - \kappa y &= ax + hy + \frac{\kappa u}{n} \\ \frac{d^2y}{dw^2} + \kappa \frac{dy}{dw} + 2\frac{dx}{dw} + \kappa x &= hx + by - \frac{\kappa}{n}(nR - v) \\ \frac{d^2z}{dw^2} + \kappa \frac{dz}{dw} &= -z\end{aligned}$$

where

$$\begin{aligned}\kappa &= \frac{1}{n}\left(kR^{-m} + \frac{dn}{dw}\right) \\ &= \frac{1}{n}(k + 3\kappa)R^{-m}\end{aligned}$$

and is essentially positive.

The terms independent of x and y produce a slowly varying, always small, term in the particular integral.

If now we consider a dynamical system in which

$$\begin{aligned}2R &= \dot{x}^2 + \dot{y}^2 + \dot{z}^2 + 2x\dot{y} - 2y\dot{x} + ax^2 + 2hxy + by^2 - z^2 \\ 2F &= \kappa(\dot{x}^2 + \dot{y}^2 + \dot{z}^2 + 2x\dot{y} - 2y\dot{x}),\end{aligned}$$

the equations of motion of this system are the same as the equations governing the free vibrations of Lagrange's third particle. Also κ

is small, and $\frac{d\kappa}{dt}$ is of the order of $k\frac{dR}{dt}$, and is therefore small of the second order.

This system is of the type already considered, and therefore the amplitude of each vibration varies as $\exp - \frac{1}{2} \int \kappa dt$.

Hence, in the case we are discussing each vibration has an amplitude dying down according to the law $\exp - \frac{1}{2} \int \kappa dw = \exp - \frac{1}{2} \int (k + 3\kappa) R^{-m} dt$.

It follows, then, that if the motion has ordinary stability, it is secularly stable. It is, however, known to be stable,* and is therefore secularly stable.

Ultimately, then, the third particle will settle down to a steady motion very near the point O. This is a minimum point of the constant C in Jacobi's integral, which in this case is therefore capable of diminishing as the time increases.

We cannot therefore regard the influence of a resisting medium as competent to cause a Trojan planet to be converted into either a minor planet or a satellite of Jupiter.

Notes on the Serious Effect of Suspension on the Rate of a Watch.

By J. J. Shaw.

(Communicated by Professor H. H. Turner.)

Professor Turner suggested that a curious experience which we had with a watch at Shide, in the Isle of Wight, might be of some interest to this Society; and by his request I have summarised the facts to put before you.

Owing to the loss of the wireless apparatus, and a difficulty in getting G.M.T. in the Isle of Wight, an attempt was made to check the Shide time by G.M.T. taken in Birmingham, by means of a watch which had a steady losing rate of 0.7 seconds per day. During this operation it was found that the watch had commenced to gain 15 seconds per day.

It had always been the practice to keep the watch in a fixed upright position during the night; but at Shide it was suspended from the bed-rail by its chain, and on the third morning it was seen to be oscillating as a pendulum, under the influence of its own balance wheel.

On arrival at the Observatory the watch was checked for one hour, and found to be keeping perfect time by the clock. It was

* Routh, *Proc. Lond. Math. Soc.*, 1875.

then suspended as in fig. 1, in such a manner that it could oscillate, and its rate was then found to have increased 2·3 seconds per hour.

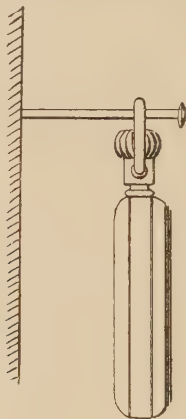


FIG. 1.

A number of observations were made during the day, and are embodied in the following table:—

	A. <i>In Pocket.</i>	B. <i>Oscillating.</i>
	Duration of Observation.	Rate in Seconds per Hour.
A.	22·36–23·36	0·0
B.	23·36–0·20	+2·2
	0·20–1·3	+2·5
	1·3–1·36	+2·2
	Average Increase	+2·3
A.	1·36–9·45	+0·02

Professor Turner's watch was tested under similar conditions, and its change of rate was as follows:—

	Duration of Observation.	Rate in Seconds per Hour.
A.	6·14–22·15	+0·04
B.	23·0–23·30	+7·8
	23·30–0·5	+6·8
	0·5–1·0	+8·4
	Average Increase	+7·5

It will be seen there is a wide difference between the changes of rate of the two watches, one being more than three times the amount of the other.

The experiments unmistakably showed how important it is *what is done with a watch during the night.*

At West Bromwich many experiments have since been made

to discover the conditions which cause the change in rate. A number of watches were tested, and all were found to be more or less affected in the same manner. The oscillations were invariably intermittent. My own watch made eight beats and stopped—the number eight being evidently due to the relation of the period of the watch acting as a pendulum and the period of the balance wheel.

It was determined to trace the effect on the rate of altering the former period; and for this purpose a light metal clip was made to hold the watch, fitted with two adjustable pivots, so that the period of the watch, acting as a pendulum, could be readily varied (fig. 2). A mirror was also fitted, and a reflected beam of light used to measure the arcual amplitude of the oscillations.

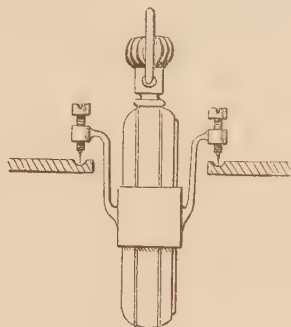


FIG. 2.

The frequency of the balance wheel was 150 per minute (the usual 18,000 train). Tests were made with the watch oscillating at various frequencies from 120 to 147. [It was found difficult to recognise frequencies close to 150 owing to the increasing synchronism deforming the true period before it was possible to determine it, and it was not convenient to stop the balance wheel.]

The following table shows for different frequencies of the watch: A, the number of beats in a synchronising period; B, the number of degrees in the oscillation; C, the increase in rate in seconds per hour:—

Watch Frequency.	A.	B.	C.	B × 5'4.
120	5'0	...	0'7	...
125	6'0	...	1'0	...
130	7'5	...	1'2	...
135	10'0	2'5	1'5	1'35
138	12'5	...	...	...
140	15'0	...	...	...
142	19'0	...	3'5	...
143	21'0	7'5	4'0	4'0
144	25'0	1'0	5'4	5'4
145	30'0	1'5	7'5	8'0
146	38'0	2'2	10'0	11'8
147	50'0	2'5	12'8	13'5

It will be seen that the rate is in close relationship to the synchronising period, and particularly to the amplitude of swing. As regards the latter, it will be seen that the rate in seconds per hour is approximately 5.4 times the amplitude in degrees. To show the close agreement, I have given, in the final column, the amplitude $\times 5.4$.

Of course, this factor 5.4 depends on the power of the mainspring and on the weight of the watch, and would vary in different watches.

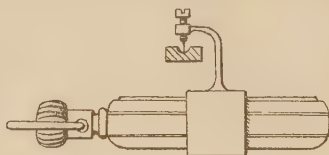


FIG. 3.

The variation between the last column and that of the rate is not more than might be expected, if consideration is given to the conditions under which the experiments were made.

So far it will be noticed the watch has consistently gained, and the nearer the frequency of the watch approached that of the balance wheel, 150, the greater the gain.

It was found impossible to get frequencies above 150, with

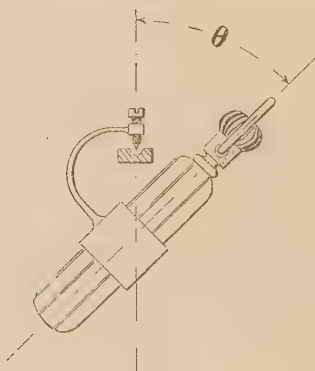


FIG. 4.

the watch oscillating in the vertical plane, for the nearer the centre of mass was brought to the axis of rotation, the more the watch became a flywheel rotating upon its axis.

The watch was suspended face upwards as in fig. 3, but in this position there was very little effect upon the rate of the watch, due to much of the oscillation being in the vertical plane, while the balance wheel was rotating in the horizontal plane.

By an adjustment of the pivot the rate could be tested in various angles, as shown in fig. 4, and frequencies as high as 180 per minute were obtainable.

The watch, however, was now *losing*, and the nearer the balance wheel was brought into the plane of the oscillations, viz. vertical, the greater the change in rate, and the more the frequency fell towards that of the balance wheel the greater the loss.

It was thought that a neutral position was reached at about 155 oscillations per minute, but at about this point the experiment becomes very critical, and the rate will suddenly change over from a maximum loss to a maximum gain and *vice versa*, without perceptible change of frequency.

The following table gives the rates for different periods and different angles, from which it is obvious that the nearer the vertical, or the nearer to 150 frequency, the greater the loss:—

Watch Frequency.	Various Inclinations.				
	20°.	25°.	30°.	35°.	45°.
180	...	- 1·8	- 0·8	...	- 0·1
174	...	...	...	...	...
168	- 3·3	- 1·8	- 1·2	...	0·0
162	- 3·7	- 2·0	- 0·8	- 0·3	- 0·1
156	- 4·7	- 3·7	...	...	...
150	- 8·2	...	...	...	...
144	+ 11·2	+ 5·3	...	...	...
138	+ 2·2	...	...	...	...

It was found impossible to test higher frequencies than 180 or smaller angles than 20°, because the point of support for limits beyond these would come within the watch case itself. Loading above the point of support was tried, but this only increased the mass and the flywheel effect came in as before and neutralised the effort.

The results obtained are, of course, in good accord with the theory of the double pendulum; but it is of interest to see them exemplified in practice; and it is of importance to remember them if good time is to be kept.

In February of this year an engine-driver, writing to the *English Mechanic*, relates some pertinent facts bearing on this subject. He says:—

“My watch has been carried some thousands of miles on a locomotive. I find that while the vibration to which it is subjected makes no difference if used on similar engines at similar speeds—*yet* a different class of engine, at a different speed, will make a lot of difference. For instance, if I am off the engine the watch will gain about 2 minutes per day. If on an engine with 5 feet 6 inches wheel, averaging 30 miles per hour, it will keep good time; while if on an engine with 7 feet 3 inches wheel at from 40 to 70 miles per hour it will gain from 60 to 90 seconds per day. I have never in my thirty years’ experience heard of an engine-driver who worked on different classes of engines say his watch kept good time.”

I think one may assume that the train going at 30 miles per hour would produce a frequency greater than that of the balance wheel; and since the speed was increased in greater proportion than the diameter of the driving wheels, if for no other reason, the higher speed should produce a higher number of oscillations. Thus the observations fall into line with the foregoing tables.

Since a railway engine can produce so great a change as 2 minutes per day—to what extent do other forms of locomotion affect our watches?

Take motoring, for instance, riding, or even walking. It seems probable that a man whose daily routine involves walking, motoring, and railway travelling, with perhaps a sedentary life on Sunday, may find that his watch has a special rate for each day of the week. The particular occupation of the wearer may be quite important, or even the rhythm of his walk will probably have its influence. It is a well-known fact in the watch trade that a watch regulated on one person may vary considerably if worn by another.

This is usually attributed to temperature, which I have always considered improbable in view of the isothermous requirements for our well-being. The rhythm in the walk, or the occupation of the wearer, active or otherwise, seem much more probable causes in view of the experiments we have just considered.

It is a common practice for watchmakers to hang up watches on hooks during the process of rating. Unless precautions are taken to prevent oscillation, this procedure seems inadvisable.

Ship's chronometers are known to have quite a different rate when in port compared with their rate at sea; so that it is necessary for an officer to know both the port rate and sea rate to enable him to make his corrections accordingly.

[Since writing the foregoing, it has been brought to my knowledge that the late Lord Kelvin made some similar experiments with watches, and communicated his conclusions in a lecture to the Institute of Engineers (Scotland) on 27th February 1867, particulars of which are given in his *Popular Lectures*, vol. ii. pp. 360–386. It is curious to note that his researches were also due to a watch of good character suddenly gaining 15 seconds in a night. His method of producing the various frequencies was by means of a bifilar suspension, and the results obtained were generally in accordance with those obtained during the recent experiments. The matter has, however, such practical importance that it may do no harm to recall attention to it.]

Note on Schjellerup's Discussion of the Occultations in the Almagest. By J. K. Fotheringham, M.A., D.Litt.

In the paper which Miss Longbottom and I contributed to *Monthly Notices* for March of this year, I began with the sentence, "Although numerous attempts have been made to determine the acceleration of the Moon's mean motion by means of ancient eclipses both of the Sun and of the Moon, I am not aware that the ancient occultations have ever been discussed." Dr. Dreyer has kindly called my attention to a paper by Schjellerup in *Copernicus*, i. (1881), pp. 223-236, entitled "*Recherches sur l'Astronomie des Anciens; III. Sur les Conjonctions d'Etoiles avec la Lune, rapportées par Ptolémée.*" In this paper Schjellerup compares the records in the *Almagest* with the lunar places as obtained from Hansen's tables and the stellar places obtained by a method of his own. He gives several interesting particulars of the positions at the times of the different observations, but does not apply them to the determination of secular acceleration. Anyone who wishes to examine the evidence would in my opinion do well to look at Schjellerup's paper as well as at that which Miss Longbottom and I have contributed.

1915 April 16.

I take this opportunity of correcting two verbal errors in our paper. On p. 392, l. 22, the word "than" should be inserted after "earlier," and on p. 393, l. 5 from the bottom, "secular acceleration" should be "centennial motion."

1915 May 12.

On the Conjunctions of Stars with the Moon, recorded by Ptolemy. By E. Nevill.

It seems to have been overlooked that these conjunctions were discussed carefully and compared with Hansen's "*Tables de la Lune*" by Dr. H. C. F. C. Schjellerup in 1881 under the title "*Recherches sur l'Astronomie des Anciens; III. Sur les Conjonctions d'Etoiles avec la Lune, rapportées par Ptolémée*" (*Copernicus*, vol. i. p. 225).

The results obtained are further discussed in my unpublished Memoir, but they appear to be too discordant to be of much value for determining the secular acceleration of the Moon.

The observed epochs of the recorded observations, as deduced by Dr. Schjellerup, would appear from my brief notes to differ materially from those given in the *Monthly Notices* for March, and this uncertainty as to the true times must still further decrease the value of the observed conjunctions.

1915 May 10.

